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For 3rd year student development economics

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Table of content

Introduction

This publication consists of lessons in Quantitative Techniques according to the ministerial program directed to third-year students in Development Economics. The content of this module is considered an essential educational tool that helps students understand the quantitative aspects of economics, with a focus on analyzing economic phenomena through models and statistical methods.

The goal of providing these lessons is to achieve the following skills:

- Understanding the definition of economic measurement and its main objectives in analyzing economic theories.

- Enabling the student to apply simple and multiple linear regression models to solve economic problems.

- Comprehending growth and development models and input-output analysis.

- Recognizing the relationship of economic measurement with other sciences such as mathematical economics and statistics.

These lessons contain five main chapters, which are:

- Chapter One: Economic Measurement.

- Chapter Two: Simple Linear Regression Analysis.

- Chapter Three: Multiple Linear Regression.

- Chapter Four: Growth and Development Model.

- Chapter Five: Input and Output Analysis.

It is our sincere hope that students find these lessons user-friendly, with clear and logical explanations, and a plethora of examples.

Chapter one

Chapter one : economic measurment

First: Definition of Economic Measurement

Economic measurement (econometric) is a branch of economics that is concerned with quantitative analysis of real economic phenomena, using appropriate statistical induction methods, that is, it is the science that uses methods of induction and statistical inference to detect objective economic laws and quantify their action.

Quantitative analysis of economic phenomena is an attempt to verify the economic relationship and ensure its logic in representing the complex reality expressed by economic theory in the form of hypotheses, and economic measurement depends on measuring and analyzing economic relations on the integration of economic theory, mathematics and statistical methods in an integrated model, with the aim of evaluating the features of that model and then choosing hypotheses about a particular economic phenomenon, and finally predicting the values of that phenomenon.

Second: Objectives of Economic Measurement

Three basic objectives of economic measurement can be identified:

2-1 Analysis and testing of various economic theories: The analysis and testing of economic theories is a major objective of economic measurement, and economic theory cannot be considered correct and acceptable unless it passes a numerical quantitative test that shows the strength of the model and explains the strength of the relationship between economic variables.

2-2 Policy formulation and decision-making: Economic measurement contributes to policy-making and decision-making by obtaining numerical values for the parameters of economic relations between variables to help governments in making current decisions in terms of providing different formulas and methods for estimating flexibility, technical parameters, marginal cost, marginal revenues, marginal tendency to consume, saving, investment, etc. At the level of the institution or state.

2-3 Predictions of the values of economic variables in the future: Economic measurement helps governments in setting policies by providing numerical values of parameters (parameter) economic variables and predicting what will be the economic phenomenon in the future, these forecasts enable policy makers and decision makers to organize economic life and take certain measures to influence certain economic variables, for example, if the government wants to know the different effects of monetary policy on inflation and unemployment, What is the expected impact of the increase in the prices of alternative or complementary goods on the required quantity of original goods, as the economic measurement will determine the level of quantity whether it is high or low and so on for the rest of the economic phenomena and what is related to the future.

Third: **The relationship of economic measurement with other sciences**

Economic measurement has a close relationship with economic theory and mathematical economics, economic statistics, and mathematical statistics, these branches are integrated in order to provide numerical values for the parameters of different economic variables, but none of these branches is a substitute for economic measurement, and this can be clarified as follows:

1- The economic theory studies the relationship between economic variables, the microeconomic theory, for example, states that the increase in the price of a commodity causes a decrease in demand for it, so this theory assumes an inverse relationship between the price and the required quantity of the commodity, but it did not give any numerical measurement of the relationship between these two variables, so it did not show the amount of decrease in the required quantity associated with a certain change in price, so this task becomes one of the tasks of economic measurement after describing it mathematically.

Thus, it can be said that the relationship between economic variables inspired by economic theory remains an abstract issue unless it is estimated in the light of realistic statistical data, which is one of the tasks of economic measurement (determining the quantitative nature of the relationships between the current economic variables in a particular reality guided by economic theory).

2- Mathematical economics is concerned with reformulating the relationship that has been determined by relying on economic theory mathematically, that is, in the form of mathematical equations and symbols without measurement or numerical proof of those formulations, measurements and proof are among the tasks of economic measurement.

3- Economic statistics Its role is limited to collecting statistical data on the economic variables that make up economic relations, recording them, scheduling or drawing them, and the role of economic measurement is to analyze and test the type of relationship between the variables in order to know the extent to which the results match with the operative economic theory.

4- The subject of mathematical statistics is equipped researcher analytical tools used in the study of relationships between economic variables and in special ways to address estimation errors in preparation for use in achieving the objectives of economic measurement.

Thus, economic metrology can be seen as the meeting point of three main sciences: economics, mathematics and statistics,

Fourth: **The Economic Model**

The model is a simple and general presentation or presentation of the complex situation that the phenomenon usually has in nature, and it reflects the basic elements that control this studied phenomenon and the mutual influence relations between them, the model is the tool used by the researcher in order to try to understand and explain the phenomena first, and then be able to estimate them and obtain expectations of their development in the future, and the difficulty of modeling lies in the need to highlight the basic elements of the phenomenon from the point of view of the problem to be interpreted by the researcher, so the phenomenon is one It can have several different models depending on the goal the researcher wants to reach and depending on the problem he wants to address.

The economic theory gives us the general theoretical basis for the nature of the phenomenon and its pattern of work and what are the basic elements that control it and the nature of the relationships between them, i.e. the form of mutual influence between them, depending on this theoretical basis, the researcher in economic measurement forms the assumed model and then estimates any calculation of its coefficients and finally divides it and from which it adopts or rejects it.

The standard models enable economic assistants and managers to make estimates and calculate quantitative forecasts for various economic variables, which helps them to make decisions and develop various economic policies effectively, economic measurement can, for example, provide a model of the quantitative relationship that links disposable income to spending on consumption, and from it we can know the potential impact of the change in the disposable income of the individual on the change in the required quantity of a commodity, as well as knowing the quantitative model of the relationship that links the price of a commodity and the quantity It is possible to calculate quantitative estimates of the elasticity of the demand for this commodity, which allows to know the relative impact of the increase in price on the required quantity of that commodity, and we can know the effect of increasing the price of a commodity on the required quantity of the substitute or complementary commodity.

Standard models also allow measuring the potential impact of an increase in the interest rate on investment, the potential impact of monetary and fiscal policy on consumption and investment and thus on inflation and unemployment.

Fifth: **Types of models**

There are several types of economic models, the most important of which are

5-1Linear and nonlinear models: This division depends on the mathematical form that the relationship between the variables of the model takes, the relationship may be of the first order (can be represented linearly) and may be in a higher form, and that relationship may take other mathematical forms such as exponential or logarithmic form.

5-2 Macro and micro models: Models can be divided according to the comprehensiveness of the model in the sense that some models include entire sectors in the national economy while others are concerned with small units in those sectors, and the model that includes the entire sectors is the total model, while that which includes small units is the micro model, and examples of macro models are the national consumption model, investment models, national income models, and foreign trade models, as for examples of partial models, production models of a particular commodity, enterprise balance models Similarly, in general, macro models are concerned with whole sectors, while micro-models are concerned with certain parts of sectors.

5.2.1 Macro models: models whose structural equations are based on the macroeconomic analysis of the variables of the national economy, for example, the total economic model of national income.

5.2.1 Micro models: They are models that address the behavior of a company, institution or part of a particular sector in the national economy, for example, the supply and demand model for a particular commodity and the short-term cost model.

5-3 Open and closed economic models: Economic models can be divided according to the extent of the participation of the national economy in international trade and its impact on it and its impact on it through the movement of exports and imports, and its role in the movement of the economy, if the model contains these two variables (or some of their parts), the economy becomes an open economy to the outside world, otherwise it is closed and the extent of the openness or closure of the economy depends on many factors, the most important of which are the economic policies followed in the state and the degree of its control and control over commodity flows. and cash to and from the rest of the world.

5.3.1Closed economic models: If the economic model includes a number of equations representing the different economic sectors without showing between them the foreign trade sector, then this model is called the closed economic model.

5.3.2 Open economic models: Macroeconomic models that take into account the foreign trade component is called open models.

5-4 Static and moving economic models: This division depends on the role played by time, whether in the formation of variables or the formation of the model itself, time may determine an important factor in the formation of the general trend in the phenomenon, it is in this case one of the external variables that enter into the formation of the model, and at other times the synchronization of variables may be different, and therefore there are slower periods in the model and may be the same in all variables in the sense that it has one impact on them, or it may be without Effect in some cases.

5.4.1 Static economic models: They are models in which time is not one of its variables or influential in changing the values of one of the variables included in it, that is, without a period of time regression, and this means that each variable has a certain value in the year in which it is located, for example, the static demand function is as follows: $D_t = f(P_t)$

5.4.2 Moving economic models: They are models in which time is one of its variables or influential in one of its variables, these models show how time affects economic variables, and these models are more realistic, for example, the moving display function is as follows: $S_t = f(P_{t-1})$

That is, the supply in the current year (t) depends on the price of the commodity in the previous year (t-1) and the kinetic variable is called a time-bound variable such as (P_{t-1}).

Sixth: **Components of the model**

The model consists of

6-1 Model equations: The economic model consists of a set of equations called these structural equations because they show the basic structure of the model to be built, and the number of equations varies from one model to another depending on the type of model and the goal of its construction and structural equations are divided into:

6.1.1Behavioral equations: They are equations that express the functional relationships between economic variables and can be expressed by a function with one independent variable or several independent variables.

6.1.2Declarative equations or identities: equations that express an economic relationship resulting from agreed definitions or are the relationship that determines the value of the dependent variable by defining it in the form of an equal relationship.

2.6 Model variables: Model equations consist of a number of variables that can be classified into several types as follows:

6.2.1Internal variables: They are the variables that affect the model and are affected by it, and their value is determined from within the model by coefficients and values of external variables, and these variables (internal) are also called dependent variables.

6.2.2 External variables: They are variables that affect the model and are not affected by it and their value is determined by factors outside the model and sometimes their value is determined by another model different from the original model and these variables are called (external) independent variables.

6.2.3 Time-bound variables: variables that belong to a previous period of time or whose values are taken from the previous period, such as (Y_{t-1}), which represents the income of the previous year.

Seventh: **Economic measurement methodology**

Chapter one : economic measurment

Economic measurement is concerned with measuring the coefficients of the model used in estimating and predicting the values of economic variables, and this requires following a certain methodology in research, because the relationship between economic variables is causal, meaning that the change in some variables has an impact on other variables, and this methodology can be determined by the following steps:

7-1 Characterization stage: The stage of characterization (formulation) of the model is one of the most important and difficult stages of building the model, through what it requires to determine the variables that must be included in the model or that must be excluded from it, and at this stage economic theory and mathematical economics are relied on to convert the mentioned relationship into mathematical equations using symbols to determine the type and direction of the relationship between economic variables, and mathematics is also relied on, such as the relationship between the required quantity of a commodity (D_x) and price (P_x) and income (Y) where the above relationship is formulated as follows: $D_x = B_0 + B_1 P_x + B_2 Y$

From the theory of demand, it is expected to obtain a negative signal for the coefficient B_1 because there is an inverse relationship between the required quantity of a commodity and its price according to economic theory and a positive sign for the coefficient B_2 because there is a direct relationship between the required quantity and consumer income.

7-2 Estimation stage: At this stage, data related to the economic phenomenon (problem) under study are collected, and then the parameters of the relationship that have been described and formulated mathematically in the first stage are estimated, i.e. the estimation of numerical values of the features (B_0, B_1, B_2) in the previous demand function, and at this stage the estimated parameters must be divided in economic, statistical and standard terms.

Economically, the values and signals of the estimated model features are compared with the values and signals related to these parameters in the light of economic theory.

- Statistically, the total and partial deviations in the variables that make up the model and the significance of the features are calculated through the tests of Student (T) and Fischer (F) and the coefficient of determination (R^2).

- In standard terms, the compatibility and fulfillment of the hypotheses of the random variable is tested on the proposed standard model, as the existence of difference means that there are problems, including the problem of self-correlation, multiplicity, and instability of homogeneity of variance, which will be discussed in detail later.

7-3 Testing stage: At this stage, the strength of the estimated model is tested by adopting certain statistical methods to ensure the validity of the model and its ability to predict, and the researcher may face here several problems, including the problem of heterogeneity of error limit, self-correlation or linear duplication and other problems, and the researcher must address these problems before starting the evaluation process.

7-4 Forecasting stage: There is no one who objects to the need to predict the future and identify it in advance before its arrival and at various macro and micro levels and in various economic and social fields and for various short, medium and long periods, and therefore at this stage future estimates are prepared for the studied variables such as the volume of demand for the commodity (D_x) But before using the estimated model in forecasting, the quality of the overall performance of the estimated model must be ascertained, and then the results reached are applied to reality and used in the forecasting process, and the research methodology in economic measurement

Second Theme: Simple Linear Regression Analysis

First: Determine the value of the model parameters

1- Definition: This model is considered one of the most common models in standard practice due to its ease of use and calculation of its parameters and applications, in addition to that, there are many economic relationships that take this form models, and the simple linear regression model is a standard model that expresses the existence of a linear relationship between two variables, one of which is the dependent variable (Y_i) and the second is the independent variable (X_i) and takes the following algebraic form:

$$Y_i = \alpha + \beta X_i + \mu_i \text{..................(01)}$$

Where:

i: Viewing number $(i = 1, 2, 3, ..., n)$

X_i : The watch value (i) of the independent variable.

α and β :Unknown community landmarks where:

:Parameter of intersection or stator, intersection of line with axis of α hierarchies Y_i

β :The coefficient of the linear model "regression coefficient" or slope indicates that if it changes X_i Bone unit the function Y_i changes by β .

μ_i :Error limit or noise or random element, for example if relation (01) refers to the consumption function Y_i In terms of disposable income X_i then μ_i It refers to various random elements other than income that go into the interpretation of the consumption function such as taste or consumer mood.

3- Analytical method for estimating the parameters of the simple linear model: We will use the most important method of estimating the parameters of the simple linear model, which is the ordinary small squares method according to the following conditions:

- The two phenomena should have a clear dialectical relationship.
- One of them is caused and the other is causing.
- The two phenomena should be measurable.

This method aims to reduce the squares of the differences between the real values and the estimated values of the dependent variable, which are the squares of the random variable $\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left(Y_i - Y_i\right)^2$

4- Estimation of the parameters of the simple linear model: It has already been mentioned that the features of the community in relation No. (01) are)α β (Unknowns, and to estimate the two parameters we follow the ordinary least squares method.

We have the equation of the regression line of society as follows:

$$Y_i = \alpha + \beta X_i + \mu_i \text{..................(01)}$$

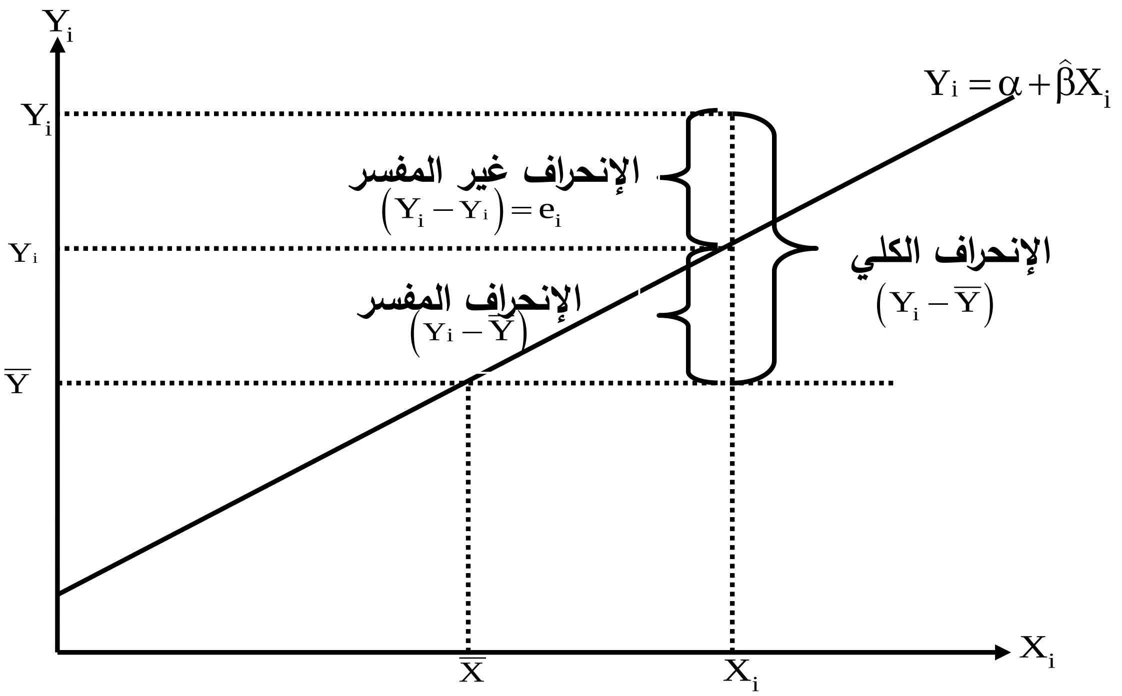
Due to the statistical difficulty of estimating the features of the population, a random sample is drawn and then the values of the features are estimated by the following equation, which is called the sample regression line as an estimate of the population regression line.

$$Y_i = \alpha + \hat{\beta} X_i + e_i \text{..................(02)}$$

To estimate Y_i independently of X_i This is done by the following equation

$$Y_i = \alpha + \hat{\beta} X_i \text{......................(03)}$$

Equation (03) is called the regression line equation and the sign (^) indicates the estimated values, not the real and each of its points Y_i represents the estimated value, and through equation (02) and (03) it is clear that the actual observations Y_i deviates from estimations Y_i by e_i As shown in the following figure:



Equation (02) can be written from equation (03) as follows:

$Y_i = Y_i + e_i$.................(04)

5- Ordinary least squares method and their properties: The ordinary least squares (OLS) method is used in estimating the slope of unknown regression, which is the method that reduces the sum of the squares of deviation of real values Y_i About Estimated Values Y_i

5-1 **Steps of the ordinary least squares (OLS) method**: Since the data values of Y_i and X_i Its spread is around the regression line, the deviation of each real value of Y_i About the estimated value of be Y_i by e_i That is

$$Y_i = Y_i + e_i \Rightarrow e_i = Y_i - Y_i$$

By squaring both sides of the equation and entering the sum on both sides, we find:

$$\sum e_i^2 = \sum \left(Y_i - Y_i\right)^2$$

And compensating for Y_i Equal to it, we find:

$$\sum e_i^2 = \sum \left(Y_i - \alpha - \hat{\beta} X_i\right)^2 \text{.....................(01)}$$

And as a mathematical condition to minimize $\sum e_i^2$ Fractional derivatives of the first order are taken for each of the α and $\hat{\beta}$ Equating each of them to zero, we find:

$$\frac{\partial \sum e_i^2}{\partial \alpha} = -02 \sum \left(Y_i - \alpha - \hat{\beta} X_i\right) = 0$$

Dividing by (-2) And decoding the parentheses we find:

$$\sum Y_i = n\alpha + \beta \sum X_i \text{...................(02)}$$

Dividing by (-2) and expanding the parentheses we find:

$$\sum X_i Y_i = \alpha \sum X_i + \beta \sum X_i^2 \text{.....................(03)}$$

Equations (02) and (03) are called the natural equations (instantaneous) where (n) the number of observations and X_i and Y_i It is always known as the values of the real observations, and once they are substituted into equations (02) and (03) and by solving them simultaneously, we get the values of (α and$\hat{\beta}$) which represent the estimations of the two real parameters (β and α).
From the two natural equations can estimate the coefficients (α and$\hat{\beta}$) of as follows:

$$\sum Y_i = n\alpha + \beta\sum X_i \text{..................}(02)$$

We have from the above

Dividing both sides of the equation by **n** we find

$$\overline{Y} = \alpha + \beta\overline{X} \Rightarrow \alpha = \overline{Y} - \beta\overline{X} \text{..................}(04)$$

We have equation (03)

$$\sum X_i Y_i = \alpha\sum X_i + \beta\sum X_i^2 \text{....................}(03)$$

We substitute (04) in (03) and we find:

$$\sum X_i Y_i = \left(\overline{Y} - \beta\overline{X}\right)\sum X_i + \beta\sum X_i^2 \Rightarrow \sum X_i Y_i = \overline{Y}\sum X_i - \beta\overline{X}\sum X_i + \beta\sum X_i^2$$

Dividing both sides of the equation by n we find

$$\frac{\sum X_i Y_i}{n} = \overline{Y}\frac{\sum X_i}{n} - \beta\overline{X}\frac{\sum X_i}{n} + \beta\frac{\sum X_i^2}{n} \Rightarrow \frac{\sum X_i Y_i}{n} = \overline{Y}\overline{X} - \beta\overline{X}\overline{X} + \beta\frac{\sum X_i^2}{n}$$

$$\Rightarrow \frac{\sum X_i Y_i}{n} = \overline{Y}\overline{X} - \beta\overline{X}^2 + \beta\frac{\sum X_i^2}{n}$$

We extract the value of the parameter β As follows

$$\frac{\sum X_iY_i}{n}-\overline{Y}\overline{X}=\beta\left(\frac{\sum X_i^2}{n}-\overline{X}^2\right)\Rightarrow\beta=\frac{\frac{\sum X_iY_i}{n}-\overline{Y}\overline{X}}{\frac{\sum X_i^2}{n}-\overline{X}^2}$$

$$\Rightarrow\beta=\frac{\sum X_iY_i-n\overline{Y}\overline{X}}{\sum X_i^2-n\overline{X}^2}$$

Therefore, the regression coefficient is the ratio between the common variance of the two variables and the variance of the independent variable, i.e.:

$$\beta=\frac{\operatorname{cov}(X_i,Y_i)}{\operatorname{var}(X_i)}$$

The regression coefficient relationship can also be written as follows:

$$\beta=\frac{\sum(X_i-\overline{X})(Y_i-\overline{Y})}{\sum(X_i-\overline{X})^2}$$

Important: To ensure that the two parameters α and $\hat{\beta}$ By the least squares method, the square of the remainder is as small as possible, we calculate the determinant of the second-order partial derivatives matrix of the estimated parameters, i.e. we extract the Hessi determinant (H) as follows:

$$|H|=\begin{bmatrix}\frac{\partial^2\sum e_i^2}{\partial\alpha^2} & \frac{\partial^2\sum e_i^2}{\partial\alpha\partial\beta}\\ \frac{\partial^2\sum e_i^2}{\partial\alpha\partial\beta} & \frac{\partial^2\sum e_i^2}{\partial\beta^2}\end{bmatrix}$$

We have from the above

$$\frac{\partial\sum e_i^2}{\partial\alpha}=-02\sum\left(Y_i-\alpha-\hat{\beta}X_i\right)=0$$

$$=-02\sum Y_i+02\sum\alpha+02\hat{\beta}\sum X_i=0$$

$$=-02\sum Y_i+02n\alpha+02\hat{\beta}\sum X_i=0$$

We partially differentiate the second time of the parameter α We find

$$\frac{\partial^2 \sum e_i^2}{\partial \alpha^2} = 02n$$

We do the partial differentiation for the second time of the two parameters α and $\hat{\beta}$ We find

$$\frac{\partial^2 \sum e_i^2}{\partial \alpha \partial \beta} = 02\sum X_i$$

We also have

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}} = -02\sum X_i \left(Y_i - \alpha - \hat{\beta} X_i \right) = 0$$

$$= -02\sum X_i Y_i + 02\alpha \sum X_i + 02\hat{\beta} \sum X_i^2 = 0$$

We partially differentiate the second time of the parameter $\hat{\beta}$ We find

$$\frac{\partial^2 \sum e_i^2}{\partial \beta^2} = 02\sum X_i^2$$

We do the partial differentiation for the second time of the two parameters α and $\hat{\beta}$ We find

$$|H| = \begin{bmatrix} \frac{\partial^2 \sum e_i^2}{\partial \alpha^2} & \frac{\partial^2 \sum e_i^2}{\partial \alpha \partial \beta} \\ \frac{\partial^2 \sum e_i^2}{\partial \alpha \partial \beta} & \frac{\partial^2 \sum e_i^2}{\partial \beta^2} \end{bmatrix} = \begin{bmatrix} 02n & 02\sum X_i \\ 02\sum X_i & 02\sum X_i^2 \end{bmatrix}$$

$$|H| = 04n \times \sum X_i^2 - 04\left(\sum X_i\right)^2 > 0$$

6. Other methods for estimating model parameters α and $\hat{\beta}$: to estimate model parameters α and $\hat{\beta}$ We use several methods, including:

Chapter two

Chapter two : Simple Linear Regression Analysis

First: Determine the value of the model parameters

It is one of the most common models in model is considered This **:Definition-1** standard practice due to its ease of use , calculation of its parameters , and applications. In addition, there are many economic relationships that take this regression model is a standard model that form of models, and the simple linear expresses the existence of a linear relationship between two variables, one of) which is the dependent variableY_i and the second It is the independent ( ) variableX_i :and takes the following algebraic form (

$$Y_i = \alpha + \beta X_i + \mu_i \text{..................}(01)$$

:where

i Viewing number : $(i = 1, 2, 3, ..., n)$

X_i) The observed value:i .of the independent variable (

α And β :Unknown community landmarks

α The parameter of the intersection or fixed part, the intersection of the line with the axis of the arrangement Y_i

β The coefficient of the linear model is the " regression coefficient " or slope, which indicates that if it changes X_i by one unit, then the function Y_i changes by an amount β .

μ_i The term of error or the element of confusion or randomness. For example, : if the relationship (01) refers to the consumption function Y_i in terms of disposable income X_i ers to various random elements other than then it ref , income that enter into the interpretation of μ_i the consumption function, such as .taste or the mood of the consumer

based The simple linear model is **:Assumptions of the simple linear model -2** :on a set of basic assumptions, which are

expectation , mathematical hope, or average of The mathematical **: hypothesis** .the residuals equals zero

$$E(\mu_i) = (Y_i - Y) = 0$$

Homogeneity of the error variance , that is , the variance or **: second hypothesis** dispersion of the residuals or the random variable σ_μ^2 is homogeneous and of values constant for alli .

$$var(\mu_i) = E(\mu_i - E(\mu_i))^2 = \sigma^2$$

variable is a continuous variable subject to a normal The random **: hypothesis** distribution, i.e $\mu_i \sim N(0, \sigma^2)$

The absence of autocorrelation between the residuals or **:Fourth hypothesis** between errors means that the common variance between the residuals is zero if i differs fromj meaning that the error generated in the first period ,i is cond periodindependent of the error generated in the sej.

$$\text{cov}(\mu_i,\mu_j) = E(\mu_i,\mu_j) = 0 \qquad \forall\, i \neq j$$

If there is a connection between them, a problem appears called the . autocorrelation problem

The values : **The fifth hypothesis** - μ_i are not related to any of the independent variables X_i that is, the absence ,of covariance and between μ_i any X_i :

$$\text{cov}(\mu_i, X_i) = E(\mu_i, X_i) = X_i E(\mu_i) = 0$$

The sample size : **sixth hypothesis**n should be much larger than the number of estimated parameters $n >>> k$

Since **:Note** μ_i it is a random variable subject to a normal distribution, and since it Y_i contains μ_i the function , Y_i has the character of randomness through the random term μ_i .i.e , $Y_i \sim N(E(Y_i), V(Y_i))$

Mathematical prediction $E(Y_i)$:

simple linear $Y_i = \alpha + \beta X_i + \mu_i$ regression equation

:By introducing the expectation on both sides of the equation, we find

$$E(Y_i) = E(\alpha) + X_i E(\beta) + E(\mu_i)$$

$$E(Y_i) = \alpha + \beta X_i \qquad ,E(\mu_i) = 0$$

Variance $\text{var}(Y_i)$ We have the following variance equation :

$$\text{var}(Y_i) = E[Y_i - E(Y_i)]^2$$

$$= E[Y_i - \alpha - \beta X_i]^2$$

$$= E[\not\alpha + \not\beta \not X_i + \mu_i - \not\alpha - \not\beta \not X_i]^2 = E[\mu_i]^2 = \sigma^2$$

permission $Y_i \sim N(\alpha + \beta X_i, \sigma^2)$

The analytical method to estimate the parameters of the simple linear -3 We will use the most important method of estimating the parameters of **:model** according to the simple linear model, which is the ordinary least squares method :the following conditions

.There should be a clear dialectical relationship between the two phenomena-

.One of them must be a result and the other caused-

.The two phenomena must be measurable

differences between the real This method aims to minimize the squares of the values and the estimated values of the dependent variable, which are the squares of the random variable $\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - Y_i)^2$

It was previously **:Estimating the parameters of the simple linear model -4**) ameters of the population in relationship No. (01) arementioned that the par α

and β are unknowns, and to estimate the two parameters we follow the (
.ordinary least squares method
: the population regression line as follows
$Y_i = \alpha + \beta X_i + \mu_i$...................(01)
Because of the statistical difficulty of estimating population parameters, a random sample is drawn and then the values of the parameters are estimated using the following equation, which is called the sample regression line as an . estimate of the population regression line
$Y_i = \alpha + \hat{\beta} X_i + e_i$...................(02)
To estimate Y_i ,independently X_i this is done using the following equation
$Y_i = \alpha + \hat{\beta} X_i$.......................(03)
Equation (03) is called the regression line equation , and the sign (^) indicates the estimated values, not the real values, and each of its points Y_i represents the estimated value. Also, through equation (02) and (03), it is clear that the actual observations Y_i deviate from the estimated values Y_i by an amount e_i .as shown

: can also be written based on equation (03) as follows (02) Equation
$Y_i = Y_i + e_i$.................(04)
The ordinary least **:The ordinary least squares method and its properties -5**
) squaresOLS the unknown regression , is used to estimate the slope of method (which is the method that reduces the sum of squares of the deviation of the true values Y_i .from the estimated values Y_i
) Steps of the Ordinary Least Squares 1-5OLS method: Since the data values L (Y_i and X_i are spread around the regression line , the deviation of each real value L Y_i from the estimated value is not Y_i by e_i .any amount
$Y_i = Y_i + e_i \Rightarrow e_i = Y_i - Y_i$
By squaring both sides of the equation and inserting the sum on both sides, we :find $\sum e_i^2 = \sum \left(Y_i - Y_i \right)^2$
Substituting for Y_i :what it is equal to, we find
$\sum e_i^2 = \sum \left(Y_i - \alpha - \hat{\beta} X_i \right)^2$ ....(01)
,As a mathematical condition for minimization $\sum e_i^2$ order -we take the first partial derivatives of both α and $\hat{\beta}$:and equate each of them to zero, we find

$$\frac{\partial \sum e_i^2}{\partial \alpha} = -02 \sum \left(Y_i - \alpha - \hat{\beta} X_i \right) = 0$$

) By dividing by-2 :and expanding the parentheses, we find ($\sum Y_i = n\alpha + \beta \sum X_i$...................(02)

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}} = -02 \sum X_i \left(Y_i - \alpha - \hat{\beta} X_i \right) = 0$$

) By dividing by-2 :and expanding the parentheses, we find ($\sum X_i Y_i = \alpha \sum X_i + \beta \sum X_i^2$.....................(03)

) Equations (02) and (03) are called the natural (simultaneous) equations, where n is the number of observations and (X_i And Y_i They are always known, given that they are the values of the true observations. Once we substitute them into) equations (02) and (03) and solve them simultaneously, we obtain the values of α and $\hat{\beta}$ (which represent estimates of the two real parameters) α and β .(

) From the two natural equations, the coefficients α and $\hat{\beta}$:As follows (

We have the above $\sum Y_i = n\alpha + \beta \sum X_i$...................(02)

By dividing both sides of the equation byn, we find $\overline{Y} = \alpha + \beta \overline{X} \Rightarrow \alpha = \overline{Y} - \beta \overline{X}$..................(04)

(We have from equation (03 $\sum X_i Y_i = \alpha \sum X_i + \beta \sum X_i^2$.....................(03)

:We substitute (04) into (03) and we find

$$\sum X_i Y_i = \left(\overline{Y} - \beta \overline{X} \right) \sum X_i + \beta \sum X_i^2 \Rightarrow \sum X_i Y_i = \overline{Y} \sum X_i - \beta \overline{X} \sum X_i + \beta \sum X_i^2$$

By dividing both sides of the equation byn, we find

$$\frac{\sum X_i Y_i}{n} = \overline{Y} \frac{\sum X_i}{n} - \beta \overline{X} \frac{\sum X_i}{n} + \beta \frac{\sum X_i^2}{n} \Rightarrow \frac{\sum X_i Y_i}{n} = \overline{Y}\overline{X} - \beta \overline{X}\overline{X} + \beta \frac{\sum X_i^2}{n}$$

$$\Rightarrow \frac{\sum X_i Y_i}{n} = \overline{Y}\overline{X} - \beta \overline{X}^2 + \beta \frac{\sum X_i^2}{n}$$

We extract the parameter value β as follows

$$\frac{\sum X_i Y_i}{n} - \overline{Y}\overline{X} = \beta \left(\frac{\sum X_i^2}{n} - \overline{X}^2 \right) \Rightarrow \beta = \frac{\frac{\sum X_i Y_i}{n} - \overline{Y}\overline{X}}{\frac{\sum X_i^2}{n} - \overline{X}^2}$$

$$\Rightarrow \beta = \frac{\sum X_i Y_i - n\overline{Y}\overline{X}}{\sum X_i^2 - n\overline{X}^2}$$

the common variance of the So the regression coefficient is the ratio between :.two variables and the variance of the independent variable, i.e

$$\beta = \frac{\text{cov}(X_i, Y_i)}{\text{var}(X_i)}$$

:regression coefficient relationship can also be written as follows

$$\beta = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{\sum (X_i - \overline{X})^2}$$

To ensure that the parameters **:Important note** α and $\hat{\beta}$ Using the least squares method, we make the square of the residuals as small as possible. We calculate order partial derivatives of the estimated -the determinant of the matrix of second) parameters, that is, we extract the Hessian determinantH : follows as (

$$|H| = \begin{bmatrix} \frac{\partial^2 \sum e_i^2}{\partial \alpha^2} & \frac{\partial^2 \sum e_i^2}{\partial \alpha \partial \beta} \\ \frac{\partial^2 \sum e_i^2}{\partial \alpha \partial \beta} & \frac{\partial^2 \sum e_i^2}{\partial \beta^2} \end{bmatrix} = \begin{bmatrix} 02n & 02\sum X_i \\ 02\sum X_i & 02\sum X_i^2 \end{bmatrix}$$

$$|H| = 04n \times \sum X_i^2 - 04\left(\sum X_i\right)^2 > 0$$

We have the above

$$\frac{\partial \sum e_i^2}{\partial \alpha} = -02\sum \left(Y_i - \alpha - \hat{\beta} X_i\right) = 0$$

$$= -02\sum Y_i + 02\sum \alpha + 02\hat{\beta}\sum X_i = 0$$

$$= -02\sum Y_i + 02n\alpha + 02\hat{\beta}\sum X_i = 0$$

We do the partial α and we differentiation for the second time for the parameter find $\frac{\partial^2 \sum e_i^2}{\partial \alpha^2} = 02n$

We do the partial differentiation a second time for the two parameters α and $\hat{\beta}$ we find $\frac{\partial^2 \sum e_i^2}{\partial \alpha \partial \beta} = 02\sum X_i$

We have it too $\frac{\partial \sum e_i^2}{\partial \hat{\beta}} = -02\sum X_i \left(Y_i - \alpha - \hat{\beta} X_i\right) = 0$

$$= -02\sum X_i Y_i + 02\alpha \sum X_i + 02\hat{\beta}\sum X_i^2 = 0$$

We do the partial $\hat{\beta}$ and we differentiation for the second time for the parameter find $\frac{\partial^2 \sum e_i^2}{\partial \beta^2} = 02\sum X_i^2$

We do the partial differentiation a second time for the two parameters α and $\hat{\beta}$

we find $\frac{\partial^2 \sum e_i^2}{\partial\alpha\partial\beta} = 02\sum X_i$

permission $|H| = \begin{bmatrix} \frac{\partial^2 \sum e_i^2}{\partial\alpha^2} & \frac{\partial^2 \sum e_i^2}{\partial\alpha\partial\beta} \\ \frac{\partial^2 \sum e_i^2}{\partial\alpha\partial\beta} & \frac{\partial^2 \sum e_i^2}{\partial\beta^2} \end{bmatrix} = \begin{bmatrix} 02n & 02\sum X_i \\ 02\sum X_i & 02\sum X_i^2 \end{bmatrix}$

$$|H| = 04n \times \sum X_i^2 - 04\left(\sum X_i\right)^2 > 0$$

) Other methods for estimating model parameters-6 α **f** $\hat{\beta}$ To estimate : **(**

) model parameters α f $\hat{\beta}$:including ,We use several methods

.Deletion and substitution method-

.Determinant method-

.Matrix method-

.(Deviation method (estimating around the origin-

From the above and from the **:Method of elimination and substitution 1-6**

simultaneous natural equations method of least squares, we have the two

By solving the two equations, we obtain the values of α **and** $\hat{\beta}$.

We have from the previous two equations **:Method of determinants 2-6**

$\sum Y_i = n\alpha + \beta\sum X_i$........................(01)

$\sum X_i Y_i = \alpha\sum X_i + \beta\sum X_i^2$...............(02)

: The previous two equations can be written in matrix form as follows

$$\begin{bmatrix} \sum Y_i \\ \sum X_i Y_i \end{bmatrix} = \begin{bmatrix} n & \sum X_i \\ \sum X_i & \sum X_i^2 \end{bmatrix} \begin{bmatrix} \alpha \\ B \end{bmatrix}$$

To estimate the values of α **and** $\hat{\beta}$ The global determinant of the matrix should calculated as follows be

$$|D| = \begin{bmatrix} n & \sum X_i \\ \sum X_i & \sum X_i^2 \end{bmatrix} = n \times \sum X_i^2 - \left(\sum X_i\right)^2$$

To find the values of α **and** $\hat{\beta}$:We use the following two equations

$$\alpha = \frac{\sum Y_i D_{11} - \sum Y_i X_i D_{12}}{n \times \sum X_i^2 - \left(\sum X_i\right)^2} \quad , \quad \beta = \frac{\sum Y_i X_i D_{22} - \sum Y_i D_{21}}{n \times \sum X_i^2 - \left(\sum X_i\right)^2}$$

where $D_{22}, D_{21}, D_{12}, D_{11}$ .are the minima of the matrix

) The following table represents the number of years of service : **Example** X_i (
) and the annual wage rate Y_i in thousands of dinars for a sample representing (
employees 08

32	28	24	20	16	12	08	04	X_i
65.80	65.00	62.60	59.00	53.90	45.40	32.70	25.60	Y_i

Find the equation of the simple regression line using the following **:Required** methods
.The least squares method-
.Determinant method-
. Deletion and substitution method-

:the solution
Before performing the calculations for the various methods, we extract the following values
$\sum X_i = 144, \sum Y_i = 410, \sum X_i Y_i = 8379.20, \sum X_i^2 = 3264, \overline{X} = 18, \overline{Y} = 51.25, n = 08$
We have the simple linear regression line **: The least squares method-01** estimated from the figure equation $Y_i = \alpha + \hat{\beta} X_i$

We know that $$\beta = \frac{\sum X_i Y_i - n\overline{X}\overline{Y}}{\sum X_i^2 - n\overline{X}^2} = \frac{8379.20 - (08)(18)(51.25)}{3264 - (08)(18)^2} = 01.4869$$

$$\alpha = \overline{Y} - \beta\overline{X} \Rightarrow \alpha = 51.25 - (01.4869)(18) = 24.4858$$

:So the estimation equation is $Y_i = \alpha + \beta X_i \Rightarrow Y_i = 24.4858 + 01.4869 X_i$
Interpretation: The estimated equation indicates that there is a direct relationship ,between the dependent variable Y_i which represents the annual wage rate, and the independent variable X_i the number of years of service. which represents ,
By increasing his job service by one year, his annual wage rate increases by .dinars 14,869
To ensure that the two parameters α and $\hat{\beta}$ Using the least squares method to square of the residuals as small as possible, we calculate the Hessian make the determinant

$$|H| = \begin{bmatrix} 02n & 02\sum X_i \\ 02\sum X_i & 02\sum X_i^2 \end{bmatrix} = \begin{bmatrix} 16 & 288 \\ 288 & 6528 \end{bmatrix}$$

$$|H| = (16)(6528) - (288)^2 = 21504 > 0$$

We have two natural equations **:Determinant method-02**

$$\begin{cases}\sum Y_i = n\alpha + B\sum X_i \\ \sum X_i Y_i = \alpha\sum X_i + B\sum X_i^2\end{cases} \Rightarrow \begin{cases}410 = 08\alpha + 144B \\ 8379.20 = 144\alpha + 3264B\end{cases}$$

: follows The previous two equations can be written in matrix form as

$$\begin{bmatrix}410 \\ 8379.20\end{bmatrix} = \begin{bmatrix}08 & 144 \\ 144 & 3264\end{bmatrix}\begin{bmatrix}\alpha \\ B\end{bmatrix}$$

To calculate a value α , β we calculate the general determinant of the matrix

$$|D| = \begin{bmatrix}08 & 144 \\ 144 & 3264\end{bmatrix} = 08 \times 3264 - (144)^2 = 5376$$

$$\alpha = \frac{\begin{bmatrix}\sum Y_i & \sum X_i \\ \sum X_i Y_i & \sum X_i^2\end{bmatrix}}{|D|} = \frac{\sum Y_i \times \sum X_i^2 - \sum X_i Y_i \times \sum X_i}{n \times \sum X_i^2 - \left(\sum X_i\right)^2} = \frac{410 \times 3264 - 8379.20 \times 144}{5376} = 24.4858$$

$$B = \frac{\begin{bmatrix}n & \sum Y_i \\ \sum X_i & \sum X_i Y_i\end{bmatrix}}{|D|} = \frac{n \times \sum X_i Y_i - \sum X_i \times \sum Y_i}{n \times \sum X_i^2 - \left(\sum X_i\right)^2} = \frac{08 \times 8379.20 - 144 \times 410}{5376} = 01.4869$$

So the estimated $Y_i = \alpha + \beta X_i \Rightarrow Y_i = 24.4858 + 01.4869 X_i$ regression equation :is

:Deletion and substitution method-03

Using the two natural equations

$$\begin{cases}\sum Y_i = n\alpha + B\sum X_i \\ \sum X_i Y_i = \alpha\sum X_i + B\sum X_i^2\end{cases} \Rightarrow \begin{cases}410 = 08\alpha + 144B ....(01) \\ 8379.20 = 144\alpha + 3264B ....(02)\end{cases}$$

We multiply equation No. (01) by 18 and subtract it from equation No. (02) to obtain a value β

$7380 = 144\alpha + 2592B$

$-$

$8379.20 = 144\alpha + 3264B$

$----------$

$-999.20 = -672B \Rightarrow B = 01.4869$

We substitute a value β :into one of the two equations and we get α

$$\begin{cases}410 = 08\alpha + 144B. \\ 8379.20 = 144\alpha + 3264B\end{cases} \Rightarrow \begin{cases}410 = 08\alpha + 144 \times 01,4869 \\ 8379.20 = 144\alpha + 3264 \times 01,4869\end{cases} \Rightarrow \begin{cases}\alpha = 24,4858 \\ \alpha = 24,4858\end{cases}$$

So the estimated $Y_i = \alpha + \beta X_i \Rightarrow Y_i = 24.4858 + 01.4869 X_i$ equation regression :is

$$\alpha = \overline{Y} - \beta\overline{X} \Rightarrow \alpha = 51,25 - 01,4869 \times 18 = 24,4858$$

.It is the same result as before

Second: Numerical features and statistical properties of the estimated parameters

that In every estimate obtained, there are several desirable characteristics for :estimate, including

.Linearity of estimators-

.Unbiased feature-

.The property of being the best estimator because it has the lowest variance-

.Covariance of parameters-

.Least mean square error-

.The estimator must be competent-

. yConsistency propert-

Least squares estimators are linear in the dependent **:Linear property-01** variable, as we note that these estimators can be described in the form of a ,function or a linear arrangement of the values of the dependent variable Y_i :.i.e

$$\beta = K_1Y_1 + K_2Y_2 + ... + K_nY_n$$

$$\beta = \sum_{i=1}^{n} K_iY_i$$

:the proof

We know that $\beta = \frac{\sum(X_i - \overline{X})(Y_i - \overline{Y})}{\sum(X_i - \overline{X})^2} = \frac{\sum x_i y_i}{\sum x_i^2}$

permission $\beta = \frac{\sum x_i (Y_i - \overline{Y})}{\sum x_i^2} = \frac{\sum x_i Y_i}{\sum x_i^2} - \frac{\overline{Y}\sum x_i}{\sum x_i^2}$

,As we know $\sum x_i = \sum(X_i - \overline{X}) = 0$ the previous relationship becomes as : follows

$$\beta = \frac{\sum x_i (Y_i - \overline{Y})}{\sum x_i^2} = \frac{\sum x_i Y_i}{\sum x_i^2}$$

the Since $\frac{x_i}{\sum x_i^2}$ valuesof $K_i = \frac{x_i}{\sum x_i^2}$

$$\beta = \sum K_i Y_i$$

$$\beta = f(Y_i)$$

So β .it's a linear expression

Characteristics of weights K_i Since the weights depend on fixed **:**X values only, they are also considered fixed and subject K_i :to the following conditions

The sum of the weights equals zero, i.e-01 $\sum K_i = 0$

We know from the above that $\sum\left(\frac{x_i}{\sum x_i^2}\right) = \frac{\sum x_i}{\sum x_i^2} = \frac{\sum(X_i - \overline{X})}{\sum x_i^2} = 0$

The sum of the product of multiplying the weights-02 K_i by the values of the independent variable X_i or by their deviations from their arithmetic mean x_i .equals the correct one, i.e $\sum K_i x_i = \sum K_i X_i = 01$

$$\sum K_i x_i = \sum K_i (X_i - \overline{X}) = \sum K_i X_i - \overline{X}\sum K_i$$

,As we know $\sum K_i = 0$: the final relationship becomes as follows

$$\sum K_i x_i = \sum K_i X_i = \sum \frac{x_i}{\sum x_i^2} \times x_i = \frac{\sum x_i^2}{\sum x_i^2} = 01$$

The sum of the squares of the weights-03 $\sum K_i^2$ is equal to the inverse of the sum of the squares of the deviations of the independent variable $\frac{01}{\sum x_i^2}$:meaning that ,

$$\sum K_i^2 = \frac{01}{\sum x_i^2} = \frac{01}{\sum\left(X_i - \overline{X}\right)^2}$$

We know from the above that $K_i = \frac{x_i}{\sum x_i^2}$ by squaring both sides of the equality :and inserting the sum, we find

$$\sum K_i^2 = \frac{\sum x_i^2}{\left(\sum x_i^2\right)^2} = \frac{01}{\sum x_i^2} = \frac{01}{\sum\left(X_i - \overline{X}\right)^2}$$

Prove that- α is a linear function of the dependent variable Y_i

$$\alpha = W_1Y_1 + W_2Y_2 + ... + W_nY_n$$

we've got

$$\alpha = \sum_{i=1}^{n} W_iY_i$$

:the proof

$$\alpha = \overline{Y} - \beta\overline{X} ....(01)$$

We know from the above that

$$\beta = \sum_{i=1}^{n} K_iY_i ....(02)$$

:Substituting (02) into (01) we find

$$\alpha = \overline{Y} - \overline{X}\sum K_iY_i \Rightarrow \alpha = \overline{Y} - \overline{X}K_i\sum Y_i$$

$$\Rightarrow \alpha = \frac{\sum Y_i}{n} - \overline{X}K_i\sum Y_i$$

$$\Rightarrow \alpha = \sum Y_i\left[\frac{01}{n} - \overline{X}K_i\right]$$

Since K_i and are $\overline{X}$ fixed quantities, we will denote them with the symbol W_i i.e

$$W_i = \left[\frac{01}{n} - \overline{X}K_i\right]$$

: So the relationship becomes as follows $\alpha = \sum Y_iW_i$

It means that the expected values for both **:bias property-The non-02** α and β and equal to $^{\text{Equal to}}\beta$ the real values α :.in society, i.e $E(\alpha) = \alpha$, $E(\beta) = \beta$

bias is that the difference between the expected -meaning of non The statistical value of this amount and the true value of the coefficient in the population is :equal to zero and is symbolized as follows

$$E(\alpha) - \alpha = 0 \;\; , E(\beta) - \beta = 0$$

bias property for-Non 01-02 β i.e **:** $E(\beta)=\beta$

We have the above

$\beta=\sum K_iY_i$......................(01)

$Y_i=\alpha+\beta X_i+\mu_i$.............(02)

:Substituting (02) into (01) we find

$\beta=\sum K_i(\alpha+\beta X_i+\mu_i)=\alpha\sum K_i+\beta\sum K_iX_i+\sum K_i\mu_i$

using weights We have the characteristics of $\sum K_i=0$, $\sum K_iX_i=01$

permission $\beta=\beta+\sum K_i\mu_i$......................(03)

By inserting the mathematical expectation on both sides of equation (03), we :find

$E(\beta)=E(\beta)+\sum K_iE(\mu_i)$

We know that $E(\mu_i)=0$

: becomes as follows So the final relationship $E(\beta)=\beta$

,Therefore β it is an unbiased estimator of the original value β

The impartiality property of 02-02 α i.e **:** $E(\alpha)=\alpha$

$\alpha=\sum Y_iW_i$.......................(01)

We know that $W_i=\left[\frac{01}{n}-\overline{X}K_i\right]$..............(02)

$Y_i=\alpha+\beta X_i+\mu_i$.............(03)

:We substitute (03) and (02) into (01) and we find

$$\alpha=\sum Y_iW_i\Rightarrow\alpha=\sum(\alpha+\beta X_i+\mu_i)\left[\frac{01}{n}-\overline{X}K_i\right]$$

$$\Rightarrow\alpha=\sum\left[\frac{01}{n}-\overline{X}K_i\right]\alpha+\sum\left[\frac{01}{n}-\overline{X}K_i\right]\beta X_i+\sum\left[\frac{01}{n}-\overline{X}K_i\right]\mu_i$$

$$\Rightarrow\alpha=\frac{n\alpha}{n}-\alpha\overline{X}\sum K_i+\frac{\beta\sum X_i}{n}-\beta\overline{X}\sum K_iX_i+\sum\left[\frac{01}{n}-\overline{X}K_i\right]\mu_i$$

$$\Rightarrow\alpha=\alpha-\alpha\overline{X}\sum K_i+\beta\overline{X}-\beta\overline{X}\sum K_iX_i+\sum\left[\frac{01}{n}-\overline{X}K_i\right]\mu_i$$

We have the characteristics of using weights $\sum K_i=0$, $\sum K_iX_i=01$

permission $\alpha=\alpha+\sum\left[\frac{01}{n}-\overline{X}K_i\right]\mu_i$....................(04)

mathematical expectation on both sides of Equation No. (04), By inserting the :we find

$$E(\alpha)=E(\alpha)+\sum\left[\frac{01}{n}-\overline{X}K_i\right]E(\mu_i)$$

We know that $E(\mu_i) = 0$

permission $E(\alpha) = \alpha$

,Therefore α value it is an unbiased estimator of the original α

Any estimates are **:(Best estimator property (minimum variance-03** considered good estimates and better than others only if their variances are .smaller compared to other estimates extracted by any other method

Property of the best estimator (minimum variance) for 01-03 β The : variance is defined by the following statistical relationship

$$\text{var}(\hat{\beta}) = E\left[\hat{\beta} - E(\hat{\beta})\right]^2 = E\left[\beta - \beta\right]^2$$

.(We have from the previous relationship No. (03

$$\beta = \beta + \sum K_i \mu_i \Rightarrow \beta - \beta = \sum K_i \mu_i$$

By squaring both sides of the equation and inserting the expectation on both :sides, we find

$$E(\beta - \beta)^2 = E\left(\sum K_i \mu_i\right)^2$$

$$\text{var}(\beta) = E\left(K_i^2 \mu_i^2 + 02\sum_{i<j}^{n} K_i K_j \mu_i \mu_j\right)$$

$$\text{var}(\beta) = K_i^2 \times E(\mu_i^2) + 02\sum_{i<j}^{n} K_i K_j E(\mu_i \mu_j)$$

We have random variable assumptions

$$\text{var}(\mu_i) = E(\mu_i)^2 = \sigma^2$$

$$\text{cov}(\mu_i, \mu_j) = E(\mu_i, \mu_j) = 0 \qquad i=j$$

permission $\text{var}(\beta) = K_i^2 \times \sigma_\mu^2$

As we know that $\sum K_i^2 = \dfrac{01}{\sum x_i^2}$

permission $\text{var}(\beta) = \sigma_\mu^2 \times \dfrac{01}{\sum x_i^2} = \dfrac{\sigma_\mu^2}{\sum (X_i - \overline{X})^2}$

Therefore, since it β ,is subject to the normal distribution $\beta \sim N\left(\beta, \dfrac{\sigma_\mu^2}{\sum (X_i - \overline{X})^2}\right)$

The property of the best estimator (minimum variance) by 02-03 α The : variance is defined by the following statistical relationship

$$\text{var}(\alpha) = E\left[\alpha - E(\alpha)\right]^2 = E\left[\alpha - \beta\right]^2$$

:(From the above, we have through Equation No. (04

$$\alpha = \alpha + \sum\left[\frac{01}{n} - \overline{X}K_i\right]\mu_i \Rightarrow \alpha - \alpha = \sum\left[\frac{01}{n} - \overline{X}K_i\right]\mu_i$$

By squaring both sides of the equation and inserting the expectation on both :sides, we find

$$E\left(\alpha - \alpha\right)^2 = E\left(\sum\left[\frac{01}{n} - \overline{X}K_i\right]\mu_i\right)^2$$

$$\text{var}\left(\alpha\right) = E\left(\sum_{i=1}^{n}\left(\frac{01}{n} - \overline{X}K_i\right)^2 (\mu_i)^2\right) + E\left(02\sum_{i<j}^{n}\left(\frac{01}{n} - \overline{X}K_i\right)\left(\frac{01}{n} - \overline{X}K_j\right)\mu_i\mu_j\right)$$

$$\text{var}\left(\alpha\right) = \sum_{i=1}^{n}\left(\frac{01}{n} - \overline{X}K_i\right)^2 E\left(\mu_i\right)^2 + 02\sum_{i<j}^{n}\left(\frac{01}{n} - \overline{X}K_i\right)\left(\frac{01}{n} - \overline{X}K_j\right)E\left(\mu_i\mu_j\right)$$

We have random variable assumptions

$$\text{var}\left(\mu_i\right) = E\left(\mu_i\right)^2 = \sigma^2$$

$$\text{cov}\left(\mu_i, \mu_j\right) = E\left(\mu_i, \mu_j\right) = 0 \qquad i{=}j$$

$$\text{var}\left(\alpha\right) = \sigma_\mu^2 \times \sum_{i=1}^{n}\left(\frac{01}{n} - \overline{X}K_i\right)^2$$

By analyzing the quantity $\left(\frac{01}{n} - \overline{X}K_i\right)^2$:Entering the sum we find

$$\text{var}\left(\alpha\right) = \sigma_\mu^2 \times \sum_{i=1}^{n}\left(\frac{01}{n} - \overline{X}K_i\right)^2 \Rightarrow \text{var}\left(\alpha\right) = \sigma_\mu^2 \times \left(\frac{n}{n^2} - \frac{02 \times \overline{X}\sum K_i}{n} + \overline{X}^2\sum K_i^2\right)$$

We have the characteristics of using weights $\sum K_i = 0$, $\sum K_i^2 = \frac{01}{\sum x_i^2}$

$$\text{var}\left(\alpha\right) = \sigma_\mu^2 \times \left(\frac{01}{n} + \frac{\overline{X}^2}{\sum x_i^2}\right)$$

Therefore, since it α ,distribution is subject to the normal

$$\alpha \sim N\left(\alpha, \sigma_\mu^2 \times \left(\frac{01}{n} + \frac{\overline{X}^2}{\sum x_i^2}\right)\right)$$

: The last relationship of variance can be analyzed as follows **:Note**

$$\text{var}\left(\alpha\right) = \sigma_\mu^2 \times \left(\frac{01}{n} + \frac{\overline{X}^2}{\sum x_i^2}\right) \Rightarrow \text{var}\left(\alpha\right) = \sigma_\mu^2 \times \left(\frac{01}{n} + \frac{\overline{X}^2}{\sum\left(X_i - \overline{X}\right)^2}\right)$$

$$\Rightarrow \text{var}\left(\alpha\right) = \sigma_\mu^2 \times \left(\frac{\sum\left(X_i - \overline{X}\right)^2 + n\overline{X}^2}{n \times \sum\left(X_i - \overline{X}\right)^2}\right)$$

: We analyze the numerator as follows

$$\sum\left(X_i-\overline{X}\right)^2+n\overline{X}^2=\sum\left(X_i^2-02X_i\overline{X}+\overline{X}^2\right)+n\overline{X}^2$$

$$=\sum X_i^2-02\overline{X}\sum X_i+n\overline{X}^2+n\overline{X}^2\Rightarrow\sum X_i^2-\cancel{02n\overline{X}^2}+\cancel{02n\overline{X}^2}$$

$$=\sum X_i^2$$

permission $\operatorname{var}\left(\alpha\right)=\sigma_\mu^2\times\left(\frac{\sum X_i^2}{n\times\sum\left(X_i-\overline{X}\right)^2}\right)$

The relationship between - $\operatorname{var}\left(\beta\right)$ **and** $\operatorname{var}\left(\alpha\right)$

We know that $\operatorname{var}\left(\alpha\right)=\sigma_\mu^2\times\left(\frac{01}{n}+\frac{\overline{X}^2}{\sum x_i^2}\right)\Rightarrow\operatorname{var}\left(\alpha\right)=\frac{\sigma_\mu^2}{n}+\overline{X}^2\times\frac{\sigma_\mu^2}{\sum x_i^2}$

$$\Rightarrow\operatorname{var}\left(\alpha\right)=\frac{\sigma_\mu^2}{n}+\overline{X}^2\times\operatorname{var}\left(\beta\right)$$

permission $\operatorname{var}\left(\alpha\right)=\frac{\sigma_\mu^2}{n}+\overline{X}^2\times\operatorname{var}\left(\beta\right)$

The covariance of the two parameters is **:Covariance of the parameters -04** given statistically as follows their covariance and is

$$\operatorname{cov}\left(\alpha,\beta\right)=E\left[\left(\alpha-\alpha\right)\left(\beta-\beta\right)\right]$$

(We have from the above equations (03) and (04

$$\beta-\beta=\sum K_i\mu_i$$

$$\alpha-\alpha=\sum W_i\mu_i$$

$$\operatorname{cov}\left(\alpha,\beta\right)=E\left[\left(\alpha-\alpha\right)\left(\beta-\beta\right)\right]\Rightarrow\operatorname{cov}\left(\alpha,\beta\right)=E\left[\left(\sum K_i\mu_i\right)\left(\sum W_i\mu_i\right)\right]$$

$$\Rightarrow\operatorname{cov}\left(\alpha,\beta\right)=E\left[\mu_i^2\sum K_iW_i\right]$$

$$\Rightarrow\operatorname{cov}\left(\alpha,\beta\right)=E\left(\mu_i^2\right)\sum K_iW_i$$

$$\Rightarrow\operatorname{cov}\left(\alpha,\beta\right)=\sigma_\mu^2\sum K_iW_i$$

We have the above $W_i=\left(\frac{01}{n}-\overline{X}K_i\right)$

$$\operatorname{cov}\left(\alpha,\beta\right)=\sigma_\mu^2\sum K_iW_i\Rightarrow\operatorname{cov}\left(\alpha,\beta\right)=\sigma_\mu^2\sum K_i\left(\frac{01}{n}-\overline{X}K_i\right)$$

$$\Rightarrow\operatorname{cov}\left(\alpha,\beta\right)=\sigma_\mu^2\left[\frac{\sum K_i}{n}-\overline{X}\sum K_i^2\right]$$

characteristics of using weights We have the $\sum K_i=0$, $\sum K_i^2=\frac{01}{\sum x_i^2}$

permission $\text{cov}(\alpha,\beta) = -\bar{X} \times \frac{\sigma_\mu^2}{\sum x_i^2} = -\bar{X} \times \frac{\sigma_\mu^2}{\sum (X_i - \bar{X})^2} = -\bar{X} \times \text{var}(\beta)$

It is derived with the smallest variance and is : **Least Mean Square Error-05** calculated as follows

$$\sigma_\mu^2 = \frac{\sum e_i^2}{n-2} = \frac{\sum (Y_i - Y_i)^2}{n-2}$$

that is, it has the characteristics of lack of **:The estimator must be efficient-06** .bias and the smallest variance at the same time compared to other estimators

We say that the parameters are consistent **:The consistency property -7** following is true estimators if the

As the sample size increases, the value of the estimated parameter approaches the value of the true parameter, and the variance of the estimated parameter :approaches zero, according to the following two conditions

$$\begin{cases} \lim\limits_{n\to\infty} E(\alpha) = \alpha \\ \lim\limits_{n\to\infty} \text{var}(\alpha) = 0 \end{cases}, \begin{cases} \lim\limits_{n\to\infty} E(\hat{\beta}) = \beta \\ \lim\limits_{n\to\infty} \text{var}(\hat{\beta}) = 0 \end{cases}$$

Third: Statistical extrapolation of the estimators of the ordinary least squares method

By knowing the distribution of α ,and $\hat{\beta}$ confidence intervals can be formed and hypotheses made about the collapse parameters can be tested α And β

Estimation with a confidence interval for the two parameters 01-03 α and β **:**

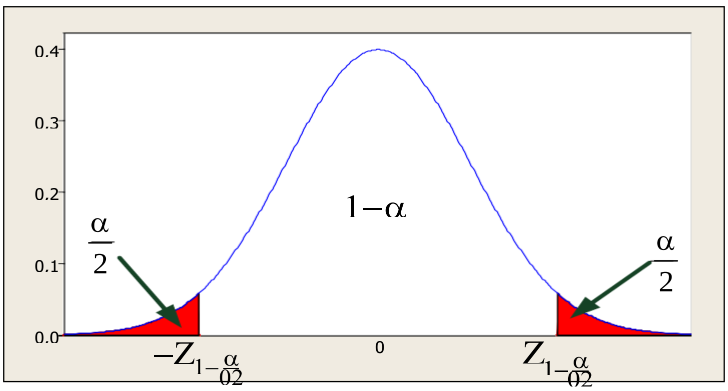
To form the confidence interval for the two parameters α ,$\hat{\beta}$ we distinguish two :cases

In the case -01 $n \geq 30$ **of** σ^2 In this case, the confidence : **known or unknown**) interval will be subject to the standard normal distribution Z for the two (parameters α and $\hat{\beta}$: as follows

$$Z = \frac{\alpha - E(\alpha)}{\sigma_\alpha} = \frac{\alpha - \alpha}{\sigma_\alpha} \sim N(0,1)$$

$$Z = \frac{\hat{\beta} - E(\hat{\beta})}{\sigma_\beta} = \frac{\hat{\beta} - \beta}{\sigma_\beta} \sim N(0,1)$$

α) based on the level of significance (The confidence interval is calculated :according to the following figure



It is clear from the figure that the confidence interval is $P\left(-Z_{1-\frac{\alpha}{2}} \leq Z \leq Z_{1-\frac{\alpha}{2}}\right)=1-\alpha$

By replacing the value of **: αConfidence range for parameter 01-01**Z in the confidence range equation, we find

$$P\left(-Z_{1-\frac{\alpha}{2}} \leq Z \leq Z_{1-\frac{\alpha}{2}}\right)=1-\alpha \Rightarrow P\left(-Z_{1-\frac{\alpha}{2}} \leq \frac{\alpha-\alpha}{\sigma_{\alpha}} \leq Z_{1-\frac{\alpha}{2}}\right)=1-\alpha$$

,By dividing both sides of the inequality by σ_{α} ,subtracting the value α and :we find ,(1-multiplying by (

$$P\left(\alpha-Z_{1-\frac{\alpha}{2}} \times \sigma_{\alpha} \leq \alpha \leq \alpha+Z_{1-\frac{\alpha}{2}} \times \sigma_{\alpha}\right)=1-\alpha$$

: The last equation is written in short as follows

$$IC(\alpha)_{1-\alpha}=\alpha \pm Z_{1-\frac{\alpha}{2}} \times \sigma_{\alpha}$$

Confidence range for parameter - 02-01**B** By replacing the value of **:**Z in the confidence range equation, we find

$$P\left(-Z_{1-\frac{\alpha}{2}} \leq Z \leq Z_{1-\frac{\alpha}{2}}\right)=1-\alpha \Rightarrow P\left(-Z_{1-\frac{\alpha}{2}} \leq \frac{\hat{\beta}-\beta}{\sigma_{\beta}} \leq Z_{1-\frac{\alpha}{2}}\right)=1-\alpha$$

,By dividing both sides of the inequality by σ_{β} ,subtracting the value $\hat{\beta}$ and :we find ,(1-multiplying by (

$$P\left(\beta-Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta} \leq \beta \leq \beta+Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta}\right)=1-\alpha$$

: The last equation is written in short as follows

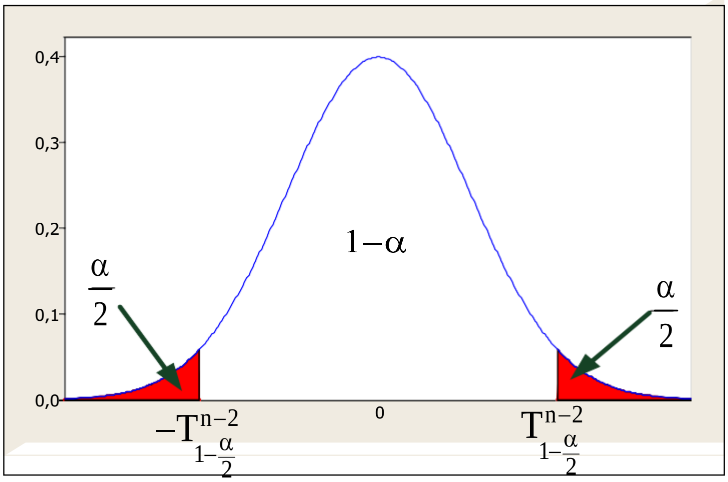
$$IC(\beta)_{1-\alpha}=\beta \pm Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta}$$

In the case of -02 $n<30$ **and** σ^2 In this case, the confidence : **is unknown**
) interval will be subject to the Student distributionT for the two parameters (α
and $\hat{\beta}$: as follows

$$T=\frac{\alpha-E(\alpha)}{\sigma_\alpha}=\frac{\alpha-\alpha}{\sigma_\alpha}\sim t\left(0,\frac{n}{n-2}\right)$$

$$T=\frac{\hat{\beta}-E(\hat{\beta})}{\sigma_\beta}=\frac{\hat{\beta}-\beta}{\sigma_\beta}\sim t\left(0,\frac{n}{n-2}\right)$$

α) The confidence interval is calculated based on the level of significance (
:according to the following figure



It is clear from the figure that the confidence interval is $P\left(-T^{n-2}_{1-\frac{\alpha}{2}}\le T\le T^{n-2}_{1-\frac{\alpha}{2}}\right)=1-\alpha$

By replacing the value of **: αConfidence range for parameter 01-02**T in the confidence range equation, we find

$$P\left(-T^{n-2}_{1-\frac{\alpha}{2}}\le T\le T^{n-2}_{1-\frac{\alpha}{2}}\right)=1-\alpha\Rightarrow P\left(-T^{n-2}_{1-\frac{\alpha}{2}}\le\frac{\alpha-\alpha}{\sigma_\alpha}\le T^{n-2}_{1-\frac{\alpha}{2}}\right)=1-\alpha$$

,By dividing both sides of the inequality by σ_α ,subtracting the value α and :we find ,(1-multiplying by (

$$P\left(\alpha-T^{n-2}_{1-\frac{\alpha}{2}}\times\sigma_\alpha\le\alpha\le\alpha+T^{n-2}_{1-\frac{\alpha}{2}}\times\sigma_\alpha\right)=1-\alpha$$

: The last equation is written in short as follows

$$IC(\alpha)_{1-\alpha}=\alpha\pm T^{n-2}_{1-\frac{\alpha}{2}}\times\sigma_\alpha$$

Confidence range for parameter 02-02B By replacing the value of **:**T in the confidence range equation, we find

$$P\left(-T^{n-2}_{1-\frac{\alpha}{2}} \le T \le T^{n-2}_{1-\frac{\alpha}{2}}\right)=1-\alpha \Rightarrow P\left(-T^{n-2}_{1-\frac{\alpha}{2}} \le \frac{\hat{\beta}-\beta}{\sigma_\beta} \le T^{n-2}_{1-\frac{\alpha}{2}}\right)=1-\alpha$$

,By dividing both sides of the inequality by σ_β ,subtracting the value $\hat{\beta}$ and :we find ,(1-multiplying by (

$$P\left(\beta - T^{n-2}_{1-\frac{\alpha}{2}} \times \sigma_\beta \le \beta \le \beta + T^{n-2}_{1-\frac{\alpha}{2}} \times \sigma_\beta\right)=1-\alpha$$

: The last equation is written in short as follows

$$IC(\beta)_{1-\alpha} = \beta \pm T^{n-2}_{1-\frac{\alpha}{2}} \times \sigma_\beta$$

Estimating the confidence interval for 02-03 σ^2_μ We can also construct the **:** to confidence interval σ^2_μ -Knowing that this field is a market subject to a chi square distribution as follows

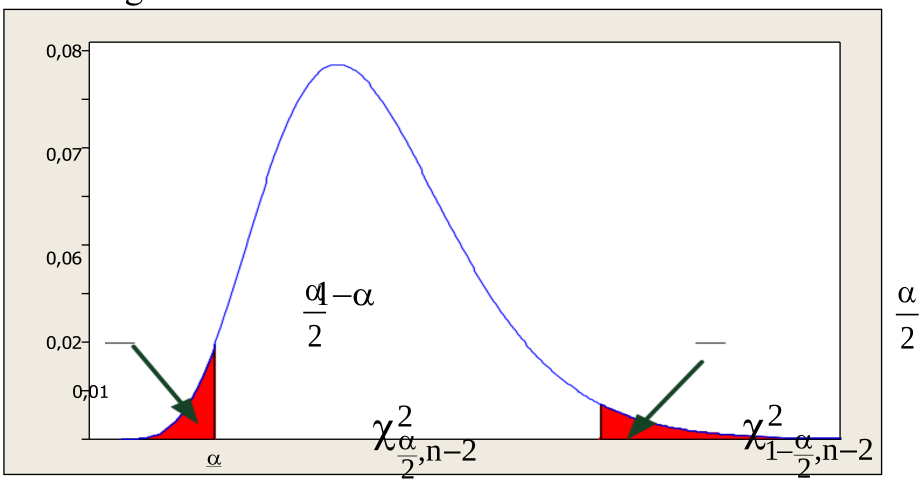
We know that
$$Z = \frac{X_i - \mu}{\sigma} \sim N(0,1)$$
$$Z^2 = \left(\frac{X_i - \mu}{\sigma}\right)^2 \sim \chi^2_1$$

As we know that $\sigma^2_\mu = \dfrac{\sum\left(Y_i - Y_i\right)^2}{(n-2)}$

Through the last relationship, multiplying both sides by the middle and dividing ,by σ^2_μ we find

$$\frac{(n-2)\times\sigma^2_\mu}{\sigma^2_\mu} = \frac{\sum\left(Y_i - Y_i\right)^2}{\sigma^2_\mu} \sim \chi^2_{\alpha,n-2}$$

following form The confidence interval takes the



It is clear from the figure that the confidence interval is

$$P\left(\chi^2_{\frac{\alpha}{2},n-2} \le \chi^2 \le \chi^2_{1-\frac{\alpha}{2},n-2}\right)=1-\alpha$$

square value-By replacing the chi χ^2 in the confidence interval equation, we :find

$$P\left(\chi^2_{\frac{\alpha}{2},n-2} \le \chi^2 \le \chi^2_{1-\frac{\alpha}{2},n-2}\right)=1-\alpha \Rightarrow P\left(\chi^2_{\frac{\alpha}{2},n-2} \le \frac{(n-2)\times\sigma_\mu^2}{\sigma_\mu^2} \le \chi^2_{1-\frac{\alpha}{2},n-2}\right)=1-\alpha$$

The confidence interval is as follows

$$P\left(\frac{(n-2)\times\hat{\sigma}_\mu^2}{\chi^2_{1-\frac{\alpha}{2},n-2}} \le \sigma_\mu^2 \le \frac{(n-2)\times\hat{\sigma}_\mu^2}{\chi^2_{\frac{\alpha}{2},n-2}}\right)=1-\alpha$$

From the same data as the previous example **:Example**

:Required

Find the confidence interval for both -1 α and β at a significance level of 05% ?

Find the confidence interval for -2 σ_μ^2 significance level of 05 a% ?

:the solution

Since the sample size $n<30$ is σ^2 unknown, the confidence interval will be) subject to a Student distributionT) with a degree of freedom (n-2 .(

Finding the confidence interval for both-01 α and β at a significance level of 05 %

Regarding the parameter 1-1 α We have the confidence interval according to : the following equation

$$P\left(\alpha - T^{n-2}_{1-\frac{\alpha}{2}}\times\sigma_\alpha \le \alpha \le \alpha + T^{n-2}_{1-\frac{\alpha}{2}}\times\sigma_\alpha\right)=1-\alpha$$

the σ_μ^2 following values first α ,, $\text{var}(\alpha)$

We know from the above that $\alpha \sim N\left(\alpha, \sigma_\mu^2 \times \left(\frac{01}{n} + \frac{\overline{X}^2}{\sum x_i^2}\right)\right)$

: We extract the value of the sum of the remainders as follows

$$\begin{cases}\sum e_i^2 = \sum\left(Y_i-\overline{Y}\right)^2 - \hat{\beta}^2\sum\left(X_i-\overline{X}\right)^2 \\ \sum e_i^2 = \sum\left(Y_i-\overline{Y}\right)^2 - \hat{\beta}\sum\left(X_i-\overline{X}\right)\left(Y_i-\overline{Y}\right)\end{cases} \Rightarrow \begin{cases}\sum e_i^2 = 1632,92-(01,4869)^2(672)=147,2048 \\ \sum e_i^2 = 1632,92-(01,4869)(999,20)=147,2048\end{cases}$$

: random error variance as follows $\sigma_\mu^2 = \frac{\sum e_i^2}{n-k} = \frac{147,2048}{08-02} = 24,53413$

Mathematical expectation and variance are as follows

$$E(\alpha)=\alpha=24,4858$$

$$\operatorname{var}(\alpha)=\begin{cases}\sigma_\mu^2\times\left(\frac{01}{n}+\frac{\overline{X}^2}{\sum x_i^2}\right)=24,53413\times\left(\frac{01}{08}+\frac{(18)^2}{672}\right)=14,896\\ \sigma_\mu^2\times\left(\frac{\sum X_i^2}{n\times\sum\left(X_i-\overline{X}\right)^2}\right)=24,53413\times\left(\frac{3264}{08\times672}\right)=14,896\end{cases}$$

We now calculate the tabulated Student value calculated at a significance level of 05%.

Student - $T_{tab}=T_{01-\frac{\alpha}{2}}^{n-k}=T_{01-\frac{\alpha}{2}}^{n-02}\Rightarrow T_{tab}=T_{0.975}^{06}=02,447$ tabular value

The trust domain is

$$P\left(\alpha-T_{1-\frac{\alpha}{2}}^{n-2}\times\sigma_\alpha\leq\alpha\leq\alpha+T_{1-\frac{\alpha}{2}}^{n-2}\times\sigma_\alpha\right)=1-\alpha\Rightarrow$$

$$P\left(24,4858-(02,447)\times\sqrt{14,896}\leq\alpha\leq24,4858-(02,447)\times\sqrt{14,896}\right)=0,95\Rightarrow$$

$$(15,0416\leq\alpha\leq33,93)=0,95$$

Regarding the parameter 2-1 β We have a confidence interval according to **:** the following equation

$$P\left(\beta-T_{1-\frac{\alpha}{2}}^{n-2}\times\sigma_\beta\leq\beta\leq\beta+T_{1-\frac{\alpha}{2}}^{n-2}\times\sigma_\beta\right)=1-\alpha$$

the σ_μ^2 following values first β ,, $\operatorname{var}(\beta)$

We know from the above that

$$\beta\sim N\left(\beta,\frac{\sigma_\mu^2}{\sum\left(X_i-\overline{X}\right)^2}\right)$$

and variance are as follows The mathematical expectation

$$E(\beta)=\beta=01,4869$$

$$\operatorname{var}(\beta)=\frac{\sigma_\mu^2}{\sum\left(X_i-\overline{X}\right)^2}=\frac{24,53413}{672}=0.03651$$

We now calculate the tabulated Student value calculated at a significance level of 05%.

Student - $T_{tab}=T_{01-\frac{\alpha}{2}}^{n-k}=T_{01-\frac{\alpha}{2}}^{n-02}\Rightarrow T_{tab}=T_{0.975}^{06}=02,447$ tabular value

The trust domain is

$$P\left(\beta - T^{n-2}_{1-\frac{\alpha}{2}} \times \sigma_{\beta} \leq \beta \leq \beta + T^{n-2}_{1-\frac{\alpha}{2}} \times \sigma_{\beta}\right) = 1-\alpha \Rightarrow$$

$$P\left(01,4869 - (02,447) \times \sqrt{0,03651} \leq \beta \leq 01,4869 + (02,447) \times \sqrt{0,03651}\right) = 0.95 \Rightarrow$$

$$P\left(01,01933 \leq \beta \leq 01,95446\right) = 0.95$$

The confidence interval is-02 σ^2_{μ} **a significance level of 05 at%** We have the **:** confidence interval equation as follows

$$P\left(\frac{(n-2)\times \hat{\sigma}^2_{\mu}}{\chi^2_{1-\frac{\alpha}{2},n-2}} \leq \sigma^2_{\mu} \leq \frac{(n-2)\times \hat{\sigma}^2_{\mu}}{\chi^2_{\frac{\alpha}{2},n-2}}\right) = 1-\alpha \Rightarrow P\left(\frac{(06)\times(24,53413)}{\chi^2_{0,975,06}} \leq \sigma^2_{\mu} \leq \frac{(06)\times(24,53413)}{\chi^2_{0,025,06}}\right) = 0.95$$

$$P\left(\frac{(06)\times(24,53413)}{14,449} \leq \sigma^2_{\mu} \leq \frac{(06)\times(24,53413)}{01,237}\right) = 0.95 \Rightarrow P\left(10,18788 \leq \sigma^2_{\mu} \leq 119\right) = 0.95$$

Fourth: Study the validity of the model using statistical and measurement tools

to study the quality and effectiveness of representing the proposed In order regression equation between (X_i, Y_i) We conduct what is called a statistical significance test . The function of this test is to ensure that the proposed l expresses well and effectively the quality of the relationship regression mode .between this test (X_i, Y_i) This test consists of several measures, the most :important of which are

: Determination coefficient and correlation coefficient 01-04

) Coefficient of determination 01-01-04 R^2 It is the ratio between the **:** (change in the estimated values over the total change. It can also be calculated via the complement with respect to the change in the residuals over the total .change

We have the above $Y_i = Y_i + e_i$

Subtracting $\overline{Y}$ from both sides of the equation and substituting e_i its equal, we :find $\left(Y_i - \overline{Y}\right) = \left(Y_i - \overline{Y}\right) + \left(Y_i - Y_i\right)$

By squaring both sides of the equality, inserting the sum and dividing byn, we :find

$$\frac{\sum\left(Y_i - \overline{Y}\right)^2}{n} = \frac{\sum\left(Y_i - \overline{Y}\right)^2}{n} + \frac{\sum\left(Y_i - Y_i\right)^2}{n}$$

SST = SSR + SSE

) Total varianceSST) explained variance = (SSR unexplained variance + ( ) or residual varianceSSE (

where

SST :Total Sum of Squares

SSR : Regression Sum of Squares
SSE : Error Sum of Squares

Another relationship for the total variance can also be found in terms of β and $\left(X_i - \overline{X}\right)$:as follows

$$Y_i = Y_i + e_i \text{.................}(01)$$

We have the above $Y_i = \alpha + \hat{\beta} X_i$.................(02)

$$\alpha = \overline{Y} - \hat{\beta}\overline{X} \text{.................}(03)$$

:Substituting (02) and (03) into (01), we find

$$Y_i = \overline{Y} - \beta\overline{X} + \hat{\beta}X_i + e_i \Rightarrow Y_i - \overline{Y} = \beta\left(X_i - \overline{X}\right) + e_i$$

:By squaring both sides of the equation and entering the sum, we find

$$\sum\left(Y_i - \overline{Y}\right)^2 = \beta^2 \sum\left(X_i - \overline{X}\right)^2 + \sum e_i^2$$

$$\sum\left(Y_i - \overline{Y}\right)^2 = \beta \sum\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right) + \sum e_i^2$$

$$SST \quad = \quad SSR \quad + \; SSE$$

So the coefficient of determination is

$$R^2 = \frac{SSR}{SST} = \frac{\sum\left(Y_i - \overline{Y}\right)^2}{\sum\left(Y_i - \overline{Y}\right)^2} = \frac{\beta^2 \sum\left(X_i - \overline{X}\right)^2}{\sum\left(Y_i - \overline{Y}\right)^2} = \frac{\hat{\beta}\sum\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sum\left(Y_i - \overline{Y}\right)^2}$$

:can also be found through the following relationship It

$$R^2 = \frac{SSR}{SST} = \frac{SST - SSE}{SST} = 01 - \frac{SSE}{SST}$$

$$= 01 - \frac{\sum\left(Y_i - \overline{Y}\right)^2}{\sum\left(Y_i - \overline{Y}\right)^2}$$

$$= 01 - \frac{\sum e_i^2}{\sum\left(Y_i - \overline{Y}\right)^2}$$

:Substituting the value of the remainders into the last equation, we find

We have the above

$$\sum e_i^2 = \sum\left(Y_i - \overline{Y}\right)^2 - \hat{\beta}^2 \sum\left(X_i - \overline{X}\right)^2$$

$$\sum e_i^2 = \sum\left(Y_i - \overline{Y}\right)^2 - \beta \sum\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)$$

:Substituting it into the previous relationship, we find

$$R^2 = 01 - \frac{\sum e_i^2}{\sum \left(Y_i - \overline{Y}\right)^2}$$

$$R^2 = \frac{\hat{\beta}^2 \sum \left(X_i - \overline{X}\right)^2}{\sum \left(Y_i - \overline{Y}\right)^2} = \frac{\hat{\beta} \sum \left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sum \left(Y_i - \overline{Y}\right)^2}$$

The value of the coefficient of determination R^2 ranges between zero and one

$$0 \leq R^2 \leq 01$$

:whereas

$R^2 = 01$ When all the diffusion points lie on the estimated $: Y_i = Y_i$ regression .line, that is , here the relationship is complete

$R^2 = 0$,Or close to it when the sample regression line is a horizontal line $Y_i = \overline{Y}$ meaning that there is no relationship between the dependent variable and the .independent variable

From the data of the previous example **:Example**

Find the coefficient of determination **:Required** R^2 ?

To calculate the coefficient of determination, we first extract the value **:Solution** of the remainders

We know from the above that

$$\sum \left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right) = \sum x_i y_i = \sum X_i Y_i - n\overline{X}\overline{Y}$$

$$= \sum x_i y_i = 8379.20 - (08)(18)(51,25) = 999.20$$

$$\sum \left(X_i - \overline{X}\right)^2 = \sum x_i^2 = \sum X_i^2 - n\overline{X}^2 \Rightarrow \sum x_i^2 = 3264 - (08)(18)^2 = 672$$

$$\sum \left(Y_i - \overline{Y}\right)^2 = \sum y_i^2 = \sum Y_i^2 - n\overline{Y}^2 \Rightarrow \sum y_i^2 = 22645,42 - (08)(51,25)^2 = 1632,92$$

$$\hat{\beta} = 01,4869$$

$$\begin{cases} \sum e_i^2 = \sum \left(Y_i - \overline{Y}\right)^2 - \hat{\beta}^2 \sum \left(X_i - \overline{X}\right)^2 \\ \sum e_i^2 = \sum \left(Y_i - \overline{Y}\right)^2 - \hat{\beta} \sum \left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right) \end{cases} \Rightarrow \begin{cases} \sum e_i^2 = 1632,92 - (01,4869)^2 (672) = 147,2048 \\ \sum e_i^2 = 1632,92 - (01,4869)(999,20) = 147,2048 \end{cases}$$

Therefore, the coefficient of determination is

$$\begin{cases} R^2 = 01 - \dfrac{\sum e_i^2}{\sum \left(Y_i - \overline{Y}\right)^2} \\ R^2 = \dfrac{\beta^2 \sum \left(X_i - \overline{X}\right)^2}{\sum \left(Y_i - \overline{Y}\right)^2} \\ R^2 = \dfrac{\hat{\beta} \sum \left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sum \left(Y_i - \overline{Y}\right)^2} \end{cases} \Rightarrow \begin{cases} R^2 = 01 - \dfrac{147,2048}{1632,92} = 0,9098 \\ R^2 = \dfrac{(01,4869)^2 (672)}{1632,92} = 0,9098 \\ R^2 = \dfrac{(01,4869)(999,20)}{1632,92} = 0,9098 \end{cases}$$

We conclude that the independent variable (number of years of **:Conclusion** service) explains about 90.98% of the change occurring in the dependent .(variable (annual wage

) Correlation coefficient 02-01-04r Correlation means the existence of a **: (** relationship between two or more phenomena, and the measure by which the degree of correlation is measured is called the correlation coefficient , which is) olsymbolized by the symbr since correlation analysis treats any two ,(variables identically and there is no distinction between the variables The dependent and the independent variable, and based on this, correlation analysis This requires the probability .assumes that both variables are random or chance of a normal distribution and can be calculated as follows, which is the square .root of the coefficient of determination, i.e $r = \pm\sqrt{R^2}$

The value of this parameter $-01 \leq r \leq +01$ varies

The connection between two or more phenomena may be positive or negative, and the sign indicates the existence of a direct or inverse relationship and does not indicate the strength of the relationship that is determined through numbers. :ng characteristics can be distinguishedThe followi

When-01 $r = +01$ This means that there is a complete and positive linear **:** relationship between the independent variable X_i and the dependent variable Y_i , meaning that an increase in the value of the variable X_i results in an increase in the value of the dependent variable Y_i .

When -02 $r = -01$ This means that there is a complete and positive linear **:** relationship between the independent variable X_i and the dependent variable Y_i , meaning that an increase in the value of the variable X_i results in a decrease in the value of the dependent variable Y_i .

When -03 $r = 0$ This means that there is no relationship between the **:** independent variable X_i and the dependent variable Y_i In this case, the . estimation of the coefficient $\hat{\beta}$) using the Ordinary Least SquaresOLS method (is equal to zero, meaning that the independent variable X_i has no effect on the dependent variable Y_i .

Same data as the previous example **:Example**

? Calculate the correlation coefficient : **Required**

Previously, we have the value of the coefficient of determination **:Solution**

$r = \pm\sqrt{R^2} \Rightarrow r = \pm\sqrt{0,9098} = \pm 0,9538$

Hypothesis testing 02-04

It is an assumption or claim (that may be true or **:Statistical hypothesis -1** incorrect) about one or more parameters for one society or for several societies. The issue of accepting this hypothesis is determined on the basis of the appropriate sample. From information obtained from the sample, so we take an the population that we are studying and we use the information of this sample to reach an appropriate decision about accepting or rejecting the statistical .hypothesis

) l hypothesisThere are two types of hypotheses, the first is called the nulThe Hypothesis) . Null and is symbolized by the symbol (H $_{0.}$ It is related to the parameter that is the subject of the study, and it specifies the values that the .researcher believes do not express the true value of the parameter

) As for the second hypothesis, it is the alternative hypothesisAlternative Hypothesis symbolized by the symbol ,(H1 , which determines the parameter values that the researcher believes are correct. Of course, the researcher hopes ll lead to the acceptance of the alternative hypothesisthat the sample data wiH1 on the basis that it is a correct hypothesis , contrary to what he hopes for the hypothesis. NothingnessH $_0$.

regression model tests , first of all, the relationship between the independent) levariab X_i) and the dependent variable (Y_i in order to verify its existence (by testing the statistical significance of the estimated parameters individually, :and the two hypotheses are formulated as follows

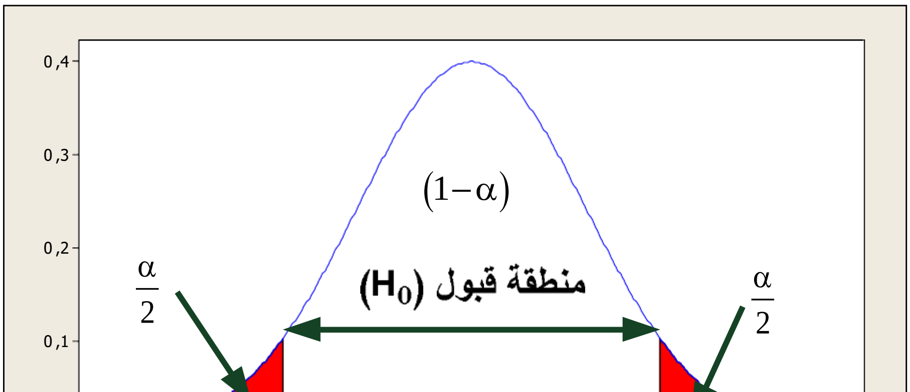
It states that there is no relationship between the two variables **:Null hypothesis-**) X_i) and (Y_i :meaning that ,(

$$H_0 : \begin{cases} \alpha = 0 \\ \beta = 0 \end{cases}$$

It states that there is a relationship between the two **:Alternative hypothesis-**) variables X_i) and (Y_i meaning that ,(

$$H_1 : \begin{cases} \alpha \neq 0 \\ \beta \neq 0 \end{cases}$$

Partial study of the model (test Student 01-02-04 T From an economic and **:** practical point of view, it is difficult for the sample size to be more than 30, especially in economics . Statistical observations for certain years and for a the ,years or sample. Therefore (15-certain variable usually range between (10) standard normal distribution testZ is rarely used and is substituted. For this (reason, resort tothe and a degree of freedomα At a significance level of . T test)nk tailed test is used, that is, on the right and left of the probability-a two ,( ) distribution. StudentT (



Knowing the sample sizen tabular the Studentα, and the significance level :value can be extracted according to the following formula

$$T_{tab} = T^{n-k}_{01-\frac{\alpha}{2}} = T^{n-02}_{01-\frac{\alpha}{2}}$$

:where

n sample size :

K the number of estimated features, in our case : α and $\hat{\beta}$

Then we calculate the calculated Student formula for each of the two parameters α as $\hat{\beta}$ follows

For the parameter α :according to the following formula $T_{cal} = \frac{\alpha - E(\alpha)}{\sigma_{\alpha}}$

For the parameter $\hat{\beta}$:according to the following formula $T_{cal} = \frac{\beta - E(\beta)}{\sigma_{\beta}}$

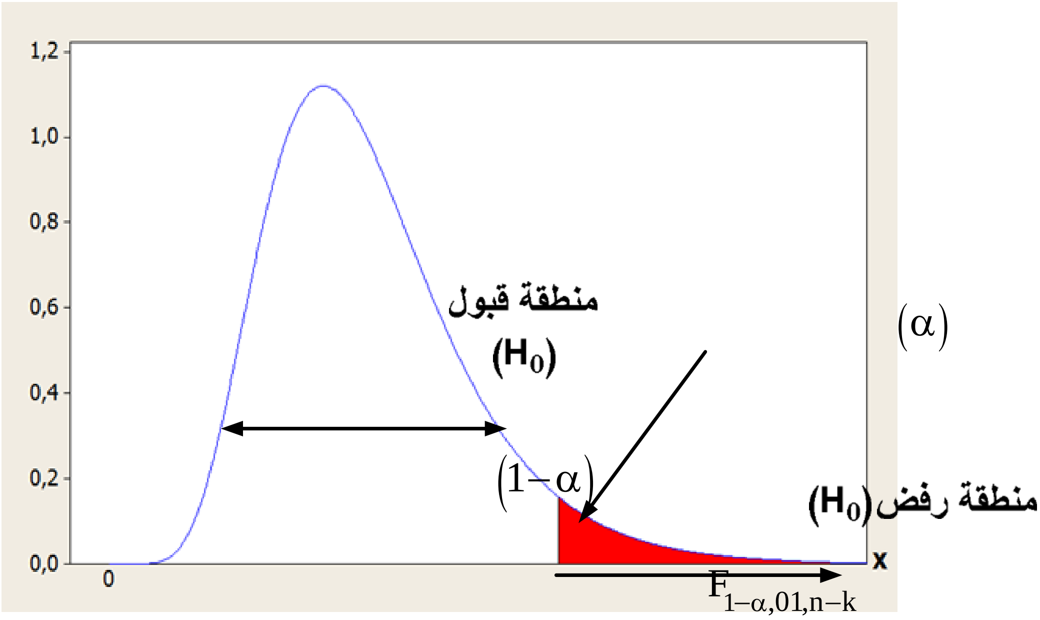
The decision will be made to reject H_0 or accept H_1 :if $T_{cal} > T_{tab}$ or $-T_{cal} < -T_{tab}$

The decision is to accept H_0 or reject H_1 :if $-T_{tab} < T_{cal} < +T_{tab}$

Overall study of the model (Fisher 02-04-02F To conduct the overall **:(test** study of the model, the significance of the regression equation as a whole is tested using Fisher's test , which tests the significance of the independent) variable X_i) and a degree of freedomα at a significance level of ,(nk and ,( ) tailed test is used for the Fisher probability distribution-the oneF The two .(:test hypotheses are as follows

) lesIt states that the relationship between the two variab : **Null hypothesis-** X_i ( ) and Y_i is not significant or significant, meaning that ($H_0 : \beta = 0$

It states that there is a fundamental relationship **:The alternative hypothesis-** $H_1 : \beta \neq 0$) between the two variables X_i) and (Y_i meaning that ,(



Knowing the sample size
n the Fisher tabular value can be extracted ,α and the significance level :according to the following formula

$$F_{tab} = F_{01-\alpha,K-1,n-k} = F_{01-\alpha,V_1,V_2}$$

Then we calculate Fisher's formula for the parameter $\hat{\beta}$:As follows

$$F_{cal} = \frac{\frac{SSR}{K-1}}{\frac{SSE}{n-k}} = \frac{\frac{\sum\left(Y_i - \overline{Y}\right)^2}{K-1}}{\frac{\sum\left(Y_i - Y_i\right)^2}{n-k}} = \frac{(n-k)\left(\sum\left(Y_i - \overline{Y}\right)^2\right)}{(K-1)\left(\sum\left(Y_i - Y_i\right)^2\right)}$$

$$= \frac{(n-k)\hat{\beta}^2\sum\left(X_i - \overline{X}\right)^2}{(K-1)\left(\sum\left(Y_i - Y_i\right)^2\right)} = \frac{(n-k)\hat{\beta}\sum\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{(K-1)\left(\sum\left(Y_i - Y_i\right)^2\right)}$$

) The relationship between the Fisher distribution 03-02-04F and the ( ) coefficient of determination R^2 The calculated Fisher value can be : (calculated according to the following relationship in terms of the coefficient of determination

We know that $R^2 = \frac{SSR}{SST} = 01 - \frac{SSE}{SST}$ ....(01)

As we know that

$$SSR = R^2 \times SST = R^2 \times \sum\left(Y_i - \overline{Y}\right)^2 \text{ ....(02)}$$

$$SSE = \left(01 - R^2\right) \times SST = \left(01 - R^2\right) \times \sum\left(Y_i - \overline{Y}\right)^2 \text{ ....(03)}$$

Substituting (02) and (03) into Fisher's formula, we find

$$F_{cal} = \frac{\frac{SSR}{K-1}}{\frac{SSE}{n-k}} = \frac{(n-k)\times SSR}{(K-1)\times SSE} = \frac{(n-k)\times R^2 \times \sum\left(Y_i - \overline{Y}\right)^2}{(K-1)\times\left(01 - R^2\right)\times\sum\left(Y_i - \overline{Y}\right)^2} = \frac{(n-k)\times R^2}{(K-1)\times\left(01 - R^2\right)}$$

$$F_{cal} = \frac{(n-k)\times R^2}{(K-1)\times(01-R^2)}$$

:where

n sample size :

K the number of estimated features, in our case : α and $\hat{\beta}$

The decision will be made to reject H_0 or accept H_1 :if $F_{cal} > F_{tab}$

The decision is to accept H_0 or reject H_1 :if $F_{cal} < F_{tab}$

04-02-04The) relationship between the Fisher distributionF and the ( ) Student distributionT distributions can The relationship between the two **:(** be found as follows

We know from the above that

$$F_{cal} = \frac{\frac{SSR}{K-1}}{\frac{SSE}{n-k}} = \frac{\frac{\hat{\beta}^2\sum(X_i-\overline{X})^2}{(K-1)}}{\frac{\left(\sum(Y_i-Y_i)^2\right)}{(n-k)}} = \frac{\frac{\hat{\beta}^2\sum(X_i-\overline{X})^2}{(K-1)}}{\frac{\left(\sum e_i^2\right)}{(n-k)}}$$

Assuming the presence of two independent variables, the previous relationship : becomes as follows

$$F_{cal} = \frac{\frac{SSR}{K-1}}{\frac{SSE}{n-k}} = \frac{\frac{\hat{\beta}^2\sum(X_i-\overline{X})^2}{(1)}}{\frac{\sum(Y_i-Y_i)^2}{(n-2)}} = \frac{\frac{\hat{\beta}^2\sum(X_i-\overline{X})^2}{(1)}}{\frac{\sum e_i^2}{(n-2)}}$$

As we also know that

$$\sigma_\mu^2 = \frac{\sum e_i^2}{n-k} = \frac{\sum e_i^2}{n-2} \quad \wedge \quad F_{cal} = \frac{\hat{\beta}^2\sum(X_i-\overline{X})^2}{\sigma_\mu^2} \quad \wedge \quad \operatorname{var}(\beta) = \frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}$$

permission

$$F_{cal} = \frac{\hat{\beta}^2\sum(X_i-\overline{X})^2}{\sigma_\mu^2} = \frac{\hat{\beta}^2}{\frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}} = \frac{\hat{\beta}^2}{\operatorname{var}(\hat{\beta})} = \left(\frac{\hat{\beta}}{\sigma_\beta}\right)^2 = T^2$$

$$F_{cal} = T^2$$

Same data as the first example : **Example**

Study the partial and total validity of the model at a significance **:Required** level of 01% ?

:the solution

We know that the estimation equation for the **:Partial study of the model-01** model is

$Yi = \alpha + \beta X_i \Rightarrow Yi = 24.4858 + 01.4869X_i$

Regarding the parameter 01-01 α :We have the following two hypotheses **:**

) There is no relationship between the variables - X_i) and (Y_i i.e ($H_0 : \alpha = 0$

) There is a relationship between the variables - X_i) and (Y_i i.e ($H_1 : \alpha \neq 0$

We know from the above that $\alpha \sim N\left(\alpha, \sigma_\mu^2 \times \left(\frac{01}{n} + \frac{\overline{X}^2}{\sum x_i^2}\right)\right)$

: random error variance as follows $\sigma_\mu^2 = \frac{\sum e_i^2}{n-k} = \frac{147,2048}{08-02} = 24,53413$

The mathematical expectation and variance are as follows

$E(\alpha) = \alpha = 24,4858$

$$\operatorname{var}(\alpha) = \begin{cases} \sigma_\mu^2 \times \left(\frac{01}{n} + \frac{\overline{X}^2}{\sum x_i^2}\right) = 24,53413 \times \left(\frac{01}{08} + \frac{(18)^2}{672}\right) = 14,896 \\ \sigma_\mu^2 \times \left(\frac{\sum X_i^2}{n \times \sum (X_i - \overline{X})^2}\right) = 24,53413 \times \left(\frac{3264}{08 \times 672}\right) = 14,896 \end{cases}$$

We now calculate the tabulated Student value calculated at a significance level of 01%.

Student - $T_{tab} = T_{01-\frac{\alpha}{2}}^{n-k} = T_{01-\frac{\alpha}{2}}^{n-02} \Rightarrow T_{tab} = T_{0.995}^{06} = 03,707$ tabular value

Student value calculated- $T_{cal} = \frac{\alpha - E(\alpha)}{\sigma_\alpha} \Rightarrow T_{cal} = \frac{24,4858 - 0}{\sqrt{14,896}} = 06,344$

Since **:Decision-** $T_{cal} > T_{tab}$ We reject H_0 and accept H_1 any relationship) the variables between X_i) and (Y_i . (

Regarding the parameter 02-01 $\hat{\beta}$:We have the following two hypotheses **:**

) There is no relationship between the variables - X_i) and (Y_i i.e ($H_0 : \beta = 0$

) There is a relationship between the variables - X_i) and (Y_i i.e ($H_1 : \beta \neq 0$

We know from the above that

$$\beta \sim N\left(\beta, \frac{\sigma_\mu^2}{\sum (X_i - \overline{X})^2}\right)$$

The mathematical expectation and variance are as follows

$$E\left(\beta\right)=\beta=01,4869$$

$$\text{var}\left(\beta\right)=\frac{\sigma_{\mu}^{2}}{\sum\left(X_i-\overline{X}\right)^2}=\frac{24,53413}{672}=0.03651$$

We now calculate the tabulated Student T value calculated at a significance level of 10%.

Student - $T_{tab}=T_{01-\frac{\alpha}{2}}^{n-k}=T_{01-\frac{\alpha}{2}}^{n-02}\Rightarrow T_{tab}=T_{0.995}^{06}=03,707$ T tabular value

Calculated $T_{cal}=\frac{\beta-E\left(\beta\right)}{\sigma_{\beta}}\Rightarrow T_{cal}=\frac{01,4869-0}{\sqrt{0,03651}}=07,7818$ Student T value

Since **:Decision-** $T_{cal}>T_{tab}$ We reject H_0 and accept H_1 any relationship) the variables between X_i) and (Y_i . (

a test Fisher at a significance level of **:Comprehensive study of the model-02** 01%

:We have the following two hypotheses

) It states that the relationship between the two variables : **Null hypothesis-** X_i ( ) and Y_i is not significant or significant, meaning that ($H_0:\beta=0$

It states that there is a fundamental relationship **:The alternative hypothesis-** $H_1:\beta\neq0$) between the two variables X_i) and (Y_i meaning that ,(

:tabulated test as: follows-Fisher's cross We calculate the value of

The Fisher tabular value at a 01% level of significance :is

$$F_{tab}=F_{01-\alpha,K-1,n-k}=F_{0,99,01,06}=13,75=\left(T_{0.995}^{06}\right)^2$$

:Fisher calculated value-

$$F_{cal}=\begin{cases}\dfrac{(n-k)\hat{\beta}^2\sum\left(X_i-\overline{X}\right)^2}{(K-1)\left(\sum\left(Y_i-Y_i\right)^2\right)}=\dfrac{(06)(01,4869)^2(672)}{(01)(147,2048)}=60,5566\\ \dfrac{(n-k)\hat{\beta}\sum\left(X_i-\overline{X}\right)\left(Y_i-\overline{Y}\right)}{(K-1)\left(\sum\left(Y_i-Y_i\right)^2\right)}=\dfrac{(06)(01,4869)(999,20)}{(01)(147,2048)}=60,5566\\ \dfrac{(n-k)\times R^2}{(K-1)\times\left(01-R^2\right)}=\dfrac{(06)(0,9098)}{(01)(01-0.9098)}=60.5570\end{cases}$$

Since **:Resolution** $F_{cal}>F_{tab}$ We reject H_0 and accept H_1 that there is a) the variables fundamental relationship between X_i) and (Y_i . (

)ANOVA The analysis of variance table aims to clarify the effect of **: table** (the independent variable X_i on the dependent variable. The importance of this , table increases when studying multiple regression Y_i as it is useful in knowing the effect of each of the independent variables on the dependent variable and thus adopting the variables influencing the The model and a variance analysis :table can be built in light of the following information

Source of variance	**The sum of the squares of the deviations**	**Degree of freedom**	**Mean square error**	**FisherF value**
The deviations are shown by the regression line , that is, by the independent variable X_i	$SSR = \sum\left(\hat{Y}_i - \overline{Y}\right)^2$ $= \hat{\beta}^2 \sum\left(X_i - \overline{X}\right)^2$ $= \hat{\beta} \sum\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)$	**K-01**	$MSR = \frac{SSR}{k-01}$ $= \frac{\sum\left(Y_i - \overline{Y}\right)^2}{01}$	$F_{cal} = \frac{SSR}{SSE}$ $= \frac{\sum\left(Y_i - \overline{Y}\right)^2 / K-01}{\sum\left(Y_i - Y_i\right)^2 / n-01}$
Deviations not shown	$SSE = \sum\left(Y_i - Y_i\right)^2$ $= \sum e_i^2$	**nk**	$MSE = \frac{SSE}{n-k}$ $= \frac{\sum\left(Y_i - Y_i\right)^2}{n-02}$	
Total deviations	$SST = \sum\left(Y_i - \overline{Y}\right)^2$	**n-01**		

From the data of the first example **:Example**

) Create an analysis of variance **:Required**ANOVA .table (

:the solution

Source of variance	**The sum of the squares of the deviations**	**Degree of freedom**	**Mean square error**	**FisherF value**
The deviations are shown by the regression line , that is, by the independent variable X_i	$SSR = \sum\left(Y_i - \overline{Y}\right)^2$ $=1485,7075$	**01**	$MSR = \frac{\sum\left(Y_i - \overline{Y}\right)^2}{01}$ $=1485,7075$	$F_{cal} = \frac{1485,7075}{24,53413}$ $= 60,5556$
Deviations not shown	$SSE = \sum\left(Y_i - Y_i\right)^2$ $= 147,2048$	**06**	$MSE = \frac{\sum\left(Y_i - Y_i\right)^2}{n-02}$ $= \frac{147,2048}{06} = 24,53413$	
Total deviations	$SST = \sum\left(Y_i - \overline{Y}\right)^2$ $= 1632,92$	**07**		

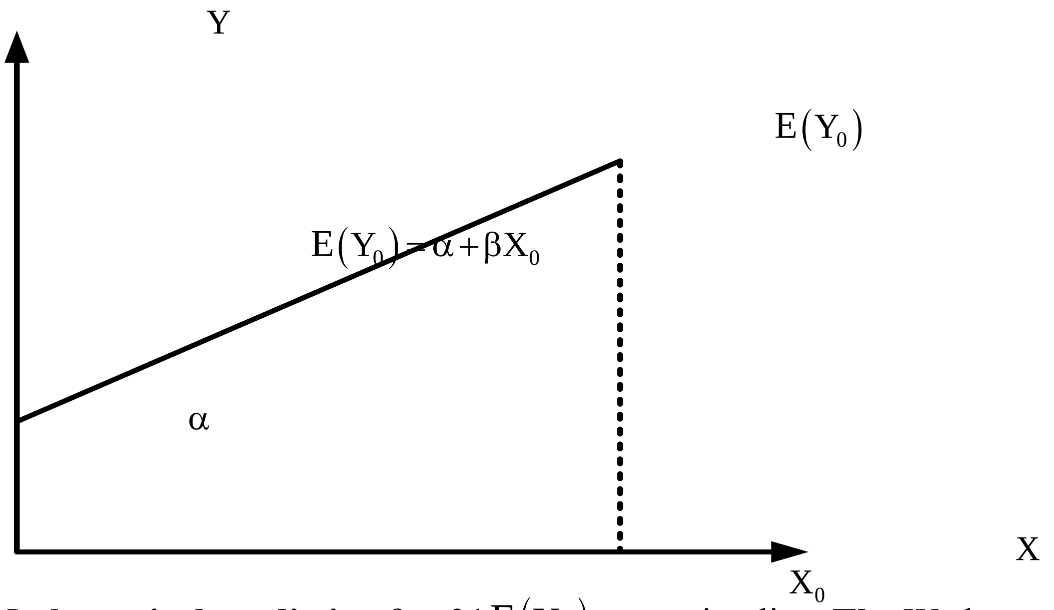
Prediction according to the simple linear regression model :Fifth

Prediction in Ordinary Least Squares models is considered one of the most important goals of this method, which is about determining the value of the dependent variable Y_i for a specific value X_i a future value or a previous value) that was not taken into account in the past when drawing the sample). Here we must differentiate between two types of Prediction, point prediction, and .prediction confidence interval

It is an expression of the expected value of the dependent **:Point forecast 01-05** ,variable $E(Y_0)=\alpha+\beta X_0$ and this is based on the following structural model $Y_i=\alpha+\beta X_i+\mu_i$ and the estimated model $Y_i=\alpha+\hat{\beta}X_i+e_i$ except that ,α it is the unknown population parameters that are estimated from the β regression line : equation for the sample as follows $Y_0=\alpha+\hat{\beta}X_0$

,Hence Y_0 it becomes an estimate for $E(Y_0)$ point estimate) and the average) value is predicted $E(Y_0)$ through the confidence interval for $E(Y_0)$ Depending on Y_0 it is necessary to calculate the mathematical expectation ,the ability, therefore and variance of the estimated value of Y_0

the estimated value Before performing the calculation, we explain $E(Y_0)$ through :the following figure



Mathematical prediction for-01 $E(Y_0)$ regression line **The** We know that Eq **:** : for the sample is as follows $Y_0=\alpha+\hat{\beta}X_0$ By introducing the mathematical :expectation on both sides of the equation, we find $E(Y_0)=E(\alpha)+E(\hat{\beta})X_0$

We know from the above that $E(\alpha)=\alpha$ and $E(\hat{\beta})=\beta$ so $E(Y_0)=\alpha+\beta X_0=E(Y_0)$

Therefore, we conclude that Y_0 it is an unbiased estimator $E(Y_0)$

Variance-02 $\operatorname{var}(Y_0)$ given by the following We know that the variance is **:** mathematical formula

$$\operatorname{var}(Y_0)=E\left[Y_0-E(Y_0)\right]^2=E\left[Y_0-E(Y_0)\right]^2$$

We also know that $Y_0=\alpha+\hat{\beta}X_0$ and $E(Y_0)=\alpha+\beta X_0=E(Y_0)$ then

$$\operatorname{var}(Y_0)=E\left[Y_0-E(Y_0)\right]^2=E\left[\alpha+\beta X_0-\alpha-\beta X_0\right]^2=E\left[(\alpha-\alpha)+X_0(\beta-\beta)\right]^2$$

:After publishing, analyzing, and entering the forecast, we find

$$\operatorname{var}(Y_0)=E\left[(\alpha-\alpha)+X_0(\beta-\beta)\right]^2=E\left[(\alpha-\alpha)^2+X_0^2(\beta-\beta)^2+02X_0(\alpha-\alpha)(\beta-\beta)\right]$$

$$=E(\alpha-\alpha)^2+X_0^2E(\beta-\beta)^2+02X_0E(\alpha-\alpha)(\beta-\beta)$$

We know from the above that

$$\operatorname{var}(\alpha)=E(\alpha-\alpha)^2$$

$$\operatorname{var}(\beta)=E(\beta-\beta)^2$$

$$\operatorname{cov}(\alpha,\beta)=E\left[(\alpha-\alpha)(\beta-\beta)\right]$$

$$=-\overline{X}\operatorname{var}(\beta)=-\overline{X}\times\frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}$$

: So, by substitution, the previous relationship becomes as follows

$$\operatorname{var}(Y_0)=E(\alpha-\alpha)^2+X_0^2E(\beta-\beta)^2+02X_0E(\alpha-\alpha)(\beta-\beta)$$

$$\operatorname{var}(Y_0)=\operatorname{var}(\alpha)+X_0^2\operatorname{var}(\beta)+02X_0\operatorname{cov}(\alpha,\beta)$$

$$\operatorname{var}(\alpha)=\sigma_\mu^2\times\left(\frac{\sum X_i^2}{n\times\sum(X_i-\overline{X})^2}\right)$$

above that As we also know from the

$$\operatorname{var}(\beta)=\frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}$$

:Substituting in the previous relationship we find

$$\operatorname{var}(Y_0)=\operatorname{var}(\alpha)+X_0^2\operatorname{var}(\beta)+02X_0\operatorname{cov}(\alpha,\beta)$$

$$\operatorname{var}(Y_0)=\sigma_\mu^2\times\left(\frac{\sum X_i^2}{n\times\sum(X_i-\overline{X})^2}\right)+X_0^2\times\frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}-02X_0\overline{X}\times\frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}$$

$$\operatorname{var}(Y_0)=\frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}\left[\frac{\sum X_i^2}{n}+X_0^2-02X_0\overline{X}\right]$$

We analyze the amount $\frac{\sum X_i^2}{n}$ as follows

Assume that $x_i=X_i-\overline{X}\Rightarrow X_i=\overline{X}+x_i$

$$\frac{\sum X_i^2}{n}=\frac{\sum(\overline{X}+x_i)^2}{n}=\frac{\sum(\overline{X}^2+02\overline{X}x_i+x_i^2)}{n}=\frac{n\overline{X}^2+02\overline{X}\sum x_i+\sum x_i^2}{n}$$

$$\frac{\sum X_i^2}{n}=\overline{X}^2+\frac{\sum x_i^2}{n}=\overline{X}^2+\frac{\sum(X_i-\overline{X})^2}{n}$$

:expression obtained in the previous relationship, we find Substituting the

$$\operatorname{var}(Y_0)=\frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}\left[\frac{\sum X_i^2}{n}+X_0^2-02X_0\overline{X}\right]\Rightarrow$$

$$\operatorname{var}(Y_0)=\frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}\left[\frac{\sum(X_i-\overline{X})^2}{n}+\overline{X}^2+X_0^2-02X_0\overline{X}\right]$$

$$\operatorname{var}(Y_0)=\frac{\sigma_\mu^2}{\sum(X_i-\overline{X})^2}\left[\frac{\sum(X_i-\overline{X})^2}{n}+(X_0-\overline{X})^2\right]$$

$$\operatorname{var}(Y_0)=\frac{\sigma_\mu^2}{n}+\frac{\sigma_\mu^2\times(X_0-\overline{X})^2}{\sum(X_i-\overline{X})^2}$$

Since the distribution Y_0) is subject to the standard normal distribution Z ,(

$$Y_0\sim N\left(\alpha+\beta X_0,\frac{\sigma_\mu^2}{n}+\frac{\sigma_\mu^2\times(X_0-\overline{X})^2}{\sum(X_i-\overline{X})^2}\right)$$

Since the unknown σ^2 its estimator are and σ^2 ,small Y_0 n is subject to the Student distribution T With the following statistics

$$T=\frac{Y_0-E(Y_0)}{\hat{\sigma}_{Y_0}}\sim t_{1-\frac{\alpha}{2}}^{n-2}$$

The confidence interval becomes as follows

$$P\left(\hat{Y}_0 - T^{n-2}_{1-\frac{\alpha}{2}} \times \sigma_{\hat{Y}_0} \le E\left(\hat{Y}_0\right) \le \hat{Y}_0 + T^{n-2}_{1-\frac{\alpha}{2}} \times \sigma_{\hat{Y}_0} \right) = 1-\alpha$$

: The last equation is written in short as follows

$$IC\left(E\left(\hat{Y}_0\right)\right)_{1-\alpha} = \hat{Y}_0 \pm T^{n-2}_{1-\frac{\alpha}{2}} \times \sigma_{\hat{Y}_0}$$

Forecasting the singular value 02-05 Y_0 Forecasting the singular value : Y_0 is only made through the estimated value $\hat{Y}_0$ **of** Y_0 It deviates from its estimated :value for two reasons

deviation - Y_0 About $E(Y_0)$ due to error μ_0

deviation - $\hat{Y}_0$ Due $E(Y_0)$ .to sampling errors

We know that $Y_0 = \alpha + \beta X_0 + \mu_0$ this $\hat{Y}_0 = \alpha + \hat{\beta} X_0$ is why we call it deviation $Y_0 - \hat{Y}_0$ With prediction error

$$Y_0 - \hat{Y}_0 = \left[\left(Y_0 - E(Y_0)\right) + \left(E(Y_0) - \hat{Y}_0\right)\right]$$

The single value is predicted Y_0 by estimating the confidence interval to which it belongs Y_0 based on the prediction error . Therefore, the mathematical , expectation and variance of the prediction error $Y_0 - \hat{Y}_0$ .must be calculated

Mathematical prediction for-01 $E\left(Y_0 - \hat{Y}_0\right)$ We have the above **:**

$Y_0 = \alpha + \beta X_0 + \mu_0$, $\hat{Y}_0 = \alpha + \hat{\beta} X_0$ then

$$E\left(Y_0 - \hat{Y}_0\right) = E\left(\alpha + \beta X_0 + \mu_0 - \alpha - \hat{\beta} X_0\right)$$

:By introducing the expectation on both sides of the equation, we find

$$E\left(Y_0 - \hat{Y}_0\right) = E\left(\alpha + \beta X_0 + \mu_0 - \alpha - \hat{\beta} X_0\right)$$

$$E\left(Y_0 - \hat{Y}_0\right) = E(\alpha) + X_0 E(\beta) + E(\mu_0) - E\left(\alpha\right) - X_0 E\left(\hat{\beta}\right)$$

$$E\left(Y_0 - \hat{Y}_0\right) = \cancel{\alpha} + \cancel{X_0 \times \beta} + 0 - \cancel{\alpha} - \cancel{X_0 \times \beta} = 0$$

permission $E\left(Y_0 - \hat{Y}_0\right) = 0$

Variance L-02 $var\left(Y_0 - \hat{Y}_0\right)$ is given by the following The variance **:** mathematical formula

$$var\left(Y_0 - \hat{Y}_0\right) = E\left[\left(Y_0 - \hat{Y}_0\right) - E\left(Y_0 - \hat{Y}_0\right)\right]^2$$

We know from the above $E\left(Y_0 - \hat{Y}_0\right) = 0$ that

$$var\left(Y_0 - \hat{Y}_0\right) = E\left[\left(Y_0 - \hat{Y}_0\right)\right]^2$$

By replacing the value Y_0 with its equivalent, as well Y_0 Equivalent to it we :find

$$\operatorname{var}\left(Y_0 - Y_0\right) = E\left[\left(Y_0 - Y_0\right)\right]^2 = E\left[\mu_0 - (\alpha - \alpha) - X_0\left(\hat{\beta} - \beta\right)\right]^2$$

We assume the amount $d_i = (\alpha - \alpha) + X_0\left(\hat{\beta} - \beta\right)$

$$\operatorname{var}\left(Y_0 - Y_0\right) = E\left[\mu_0 + d_i\right]^2 = E\left[\mu_0^2 + 02\mu_0 d_i + d_i^2\right]$$

$$= E\left(\mu_0^2\right) + 02E\left(\mu_0\right)E\left(d_i\right) + E\left(d_i^2\right)$$

$$= \operatorname{var}\left(\mu_0^2\right) + 0 + E\left(d_i^2\right)$$

$$= \sigma_\mu^2 + E\left(d_i^2\right)$$

We analyze the expression $E\left(d_i^2\right)$:and get

$$E\left(d_i^2\right) = E\left((\alpha - \alpha) + X_0\left(\hat{\beta} - \beta\right)\right)^2 = E\left[(\alpha - \alpha)^2 + 02X_0(\alpha - \alpha)\left(\hat{\beta} - \beta\right) + X_0^2\left(\hat{\beta} - \beta\right)^2\right]$$

$$= E(\alpha - \alpha)^2 + 02X_0 E(\alpha - \alpha)\left(\hat{\beta} - \beta\right) + X_0^2 E\left(\hat{\beta} - \beta\right)^2$$

$$= \operatorname{var}\left(\widehat{\alpha}\right) + 02X_0 \operatorname{cov}\left(\widehat{\alpha, \beta}\right) + X_0^2 \operatorname{var}\left(\widehat{\beta}\right)$$

We know from the above that $E\left(d_i^2\right) = \operatorname{var}(\alpha) + 02X_0 \operatorname{cov}(\alpha, \beta) + X_0^2 \operatorname{var}(\beta)$

$$= \frac{\sigma_\mu^2}{n} + \frac{\sigma_\mu^2 \times \left(X_0 - \overline{X}\right)^2}{\sum\left(X_i - \overline{X}\right)^2}$$

permission

$$\operatorname{var}\left(Y_0 - Y_0\right) = \sigma_\mu^2 + E\left(d_i^2\right)$$

$$= \sigma_\mu^2 + \frac{\sigma_\mu^2}{n} + \frac{\sigma_\mu^2 \times \left(X_0 - \overline{X}\right)^2}{\sum\left(X_i - \overline{X}\right)^2}$$

$$= \sigma_\mu^2\left[01 + \frac{01}{n} + \frac{\left(X_0 - \overline{X}\right)^2}{\sum\left(X_i - \overline{X}\right)^2}\right]$$

Therefore, the discrepancy is $\operatorname{var}\left(Y_0 - Y_0\right) = \sigma_\mu^2\left[01 + \frac{01}{n} + \frac{\left(X_0 - \overline{X}\right)^2}{\sum\left(X_i - \overline{X}\right)^2}\right]$

Since the distribution $\left(Y_0 - Y_0\right)$) is subject to the standard normal distributionZ ,(

$$\left(Y_0 - Y0\right) \sim N\left(0, \sigma_\mu^2\left[01 + \frac{01}{n} + \frac{\left(X_0 - \overline{X}\right)^2}{\sum\left(X_i - \overline{X}\right)^2}\right]\right)$$

It is estimated σ^2 by calculating the variance of the residuals for σ^2 n ,small and) and the estimate (for the variance error) follows the Student distributionT (

: with the following statistics $T = \frac{\left(Y_0 - Y0\right) - E\left(Y_0 - Y0\right)}{\hat{\sigma}\left(Y_0 - Y0\right)} \sim t_{1-\frac{\alpha}{2}}^{n-2}$

: The confidence interval is given as follows

$$P\left(-T_{1-\frac{\alpha}{2}}^{n-2} \le T \le T_{1-\frac{\alpha}{2}}^{n-2}\right) = 1 - \alpha \Rightarrow P\left(-T_{1-\frac{\alpha}{2}}^{n-2} \le \frac{\left(Y_0 - Y0\right) - E\left(Y_0 - Y0\right)}{\sigma\left(Y_0 - Y0\right)} \le T_{1-\frac{\alpha}{2}}^{n-2}\right) = 1 - \alpha$$

follows The confidence interval becomes as

$$P\left(Y0 - T_{1-\frac{\alpha}{2}}^{n-2} \times \sigma\left(Y_0 - Y0\right) \le Y_0 \le Y0 + T_{1-\frac{\alpha}{2}}^{n-2} \times \sigma\left(Y_0 - Y0\right)\right) = 1 - \alpha$$

:The abbreviation is written as follows

$$IC\left(Y_0\right)_{1-\alpha} = Y0 \pm T_{1-\frac{\alpha}{2}}^{n-2} \times \sigma\left(Y_0 - Y0\right)$$

Chapter three

Chapter three : Multiple Linear Regression

:Introduction -First

The general linear model is considered an extension of the classical (simple) linear model, because the simple model studies the relationship between the dependent variable Y_i The only independent variable X_i while the general , linear model considers that a variable Y_i is dependent on more than one .(independent variable (at least two independent variables

there is a linear The multiple linear model is based on the assumption that relationship between a dependent variable Y_i A number of independent variables $(X_1, X_2, X_3, ..., X_K)$ and a random term μ_i This relationship is expressed for n observations andK :independent variables as follows

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + ... + \beta_k X_{ik} + \mu_i$$

In fact, this equation is one of a number ofn equations that form a system of the :following equations

$$Y_1 = \beta_0 + \beta_1 X_{11} + \beta_2 X_{12} + \beta_3 X_{13} + ... + \beta_k X_{1k} + \mu_1$$
$$Y_2 = \beta_0 + \beta_1 X_{21} + \beta_2 X_{22} + \beta_3 X_{23} + ... + \beta_k X_{2k} + \mu_2$$
$$Y_3 = \beta_0 + \beta_1 X_{31} + \beta_2 X_{32} + \beta_3 X_{33} + ... + \beta_k X_{3k} + \mu_3$$
$$. \;=\; . \quad . \quad . \quad . \; . \; . \; . \; . \; . \; . \; .$$
$$. \;=\; . \quad . \quad . \quad . \; . \; . \; . \; . \; . \; . \; .$$
$$. \;=\; . \quad . \quad . \quad . \; . \; . \; . \; . \; . \; . \; .$$
$$Y_n = \beta_0 + \beta_1 X_{n1} + \beta_2 X_{n2} + \beta_3 X_{n3} + ... + \beta_k X_{nk} + \mu_n$$

:These equations can be formulated in matrix form as follows

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ . \\ . \\ . \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{12} & X_{13} & . & . & . & X_{1K} \\ 1 & X_{21} & X_{22} & X_{23} & . & . & . & X_{2K} \\ 1 & X_{31} & X_{32} & X_{33} & . & . & . & X_{3K} \\ 1 & . & . & . & . & . & . & . \\ 1 & . & . & . & . & . & . & . \\ 1 & . & . & . & . & . & & . \\ 1 & X_{n1} & X_{n2} & X_{n3} & . & . & . & X_{nk} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ . \\ . \\ \beta_k \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ . \\ . \\ . \\ \mu_n \end{bmatrix}$$

:Write an abbreviation as follows $Y = XB + \mu$

:where

Y order vertical vector-A rank :$(n \times 1)$ containingn observations of the dependent variable Y_i

X ,A matrix of observations of independent variables of rank $(n \times k)$ and its first .column contains the values of the correct one

B A vertical vector of rank: $(k\times1)$ containing the unknown features, i.e $(\beta_0,\beta_1,\beta_2,\beta_3,..........,\beta_k)$

μ A vertical vector of rank : $(n\times1)$ containing the values of the μ_i unknown .random variable

multiple linear model Second: Assumptions of the

) When using the Ordinary Least SquaresOLS method to estimate the multiple (:linear regression model, a set of assumptions must be met, as follows

We assume that there is a linear relationship between the dependent variable **-01** Y_i And independent variables $(X_1,X_2,X_3,............,X_K)$ In the presence of random error μ_i :which reflects the rest of the variables and errors, that is ,

$$Y_{N\times1}=X_{N\times K}B_{K\times1}+\mu_{N\times1}$$

The expected value of the error term vector is equal to zero **-02** $E(\mu_i)=0$ This . means that the expected value of each element of the random vector μ_i equals :. zero, i.e

$$E(\mu_i)=E\begin{bmatrix}\mu_1\\\mu_2\\\mu_3\\.\\.\\.\\\mu_n\end{bmatrix}=\begin{bmatrix}E(\mu_1)\\E(\mu_2)\\E(\mu_3)\\.\\.\\.\\E(\mu_n)\end{bmatrix}=\begin{bmatrix}0\\0\\0\\.\\.\\.\\0\end{bmatrix}=0$$

The variance of the random elements is constant and equal **-03** σ^2 and the , :.common variance between them is equal to zero, i.e

$$\text{var}(\mu_i)=E(\mu_i)^2=E(\mu\times\mu^T)=\sigma_\mu^2 I_n$$

$$\text{cov}(\mu_i,\mu_j)=E(\mu_i,\mu_j)=0 \qquad \forall i\neq j$$

:whereas

I_n It is a unit matrix of order :n diagonal is one and the rest of the its main) .(elements are zero

The previous two hypotheses can be clarified through a matrix of variances and :covariances of errors as follows

$$\operatorname{var}(\mu_i)=E(\mu_i)^2=E(\mu\times\mu^T)$$

$$=E\begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\\ .\\ .\\ .\\ \mu_n\end{bmatrix}\times\begin{bmatrix}\mu_1 & \mu_2 & \mu_3 & . & . & . & \mu_n\end{bmatrix}=E\begin{bmatrix}\mu_1^2 & \mu_1\mu_2 & \mu_1\mu_3 & \mu_1\mu_4 & . & . & . & \mu_1\mu_n\\ \mu_2\mu_1 & \mu_2^2 & \mu_2\mu_3 & \mu_2\mu_4 & . & . & . & \mu_2\mu_n\\ \mu_3\mu_1 & \mu_3\mu_2 & \mu_3^2 & \mu_3\mu_4 & . & . & . & \mu_3\mu_n\\ . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & & .\\ \mu_n\mu_1 & \mu_n\mu_2 & \mu_n\mu_3 & \mu_n\mu_4 & . & . & . & \mu_n^2\end{bmatrix}$$

$$=\begin{bmatrix}E(\mu_1^2) & E(\mu_1\mu_2) & E(\mu_1\mu_3) & E(\mu_1\mu_4) & . & . & . & E(\mu_1\mu_n)\\ E(\mu_2\mu_1) & E(\mu_2^2) & E(\mu_2\mu_3) & E(\mu_2\mu_4) & . & . & . & E(\mu_2\mu_n)\\ E(\mu_3\mu_1) & E(\mu_3\mu_2) & E(\mu_3^2) & E(\mu_3\mu_4) & . & . & . & E(\mu_3\mu_n)\\ . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & .\\ E(\mu_n\mu_1) & E(\mu_n\mu_2) & E(\mu_n\mu_3) & E(\mu_n\mu_4) & . & . & . & E(\mu_n^2)\end{bmatrix}$$

$$=\begin{bmatrix}\operatorname{var}(\mu_1^2) & \operatorname{cov}(\mu_1\mu_2) & \operatorname{cov}(\mu_1\mu_3) & \operatorname{cov}(\mu_1\mu_4) & . & . & . & \operatorname{cov}(\mu_1\mu_n)\\ \operatorname{cov}(\mu_2\mu_1) & \operatorname{var}(\mu_2^2) & \operatorname{cov}(\mu_2\mu_3) & \operatorname{cov}(\mu_2\mu_4) & . & . & . & \operatorname{cov}(\mu_2\mu_n)\\ \operatorname{cov}(\mu_3\mu_1) & \operatorname{cov}(\mu_3\mu_2) & \operatorname{var}(\mu_3^2) & \operatorname{cov}(\mu_3\mu_4) & . & . & . & \operatorname{cov}(\mu_3\mu_n)\\ . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & .\\ \operatorname{cov}(\mu_n\mu_1) & \operatorname{cov}(\mu_n\mu_2) & \operatorname{cov}(\mu_n\mu_3) & \operatorname{cov}(\mu_n\mu_4) & . & . & . & \operatorname{var}(\mu_n^2)\end{bmatrix}$$

$$=\begin{bmatrix}\sigma_1^2 & 0 & 0 & 0\\ 0 & \sigma_2^2 & 0 & 0\\ 0 & 0 & \sigma_3^2 & 0\\ 0 & 0 & 0 & \sigma_n^2\end{bmatrix}=\sigma^2\times I_n$$

whereas$\left(\sigma_1^2=\sigma_2^2=\sigma_3^2=\sigma_4^2=..........=\sigma_n^2\right)$

The random variable **-04** μ_i follows a normal distribution, that is $\mu_i\sim N(0,\sigma_\mu^2)$

The data matrix **-05** X random matrix, meaning it contains fixed -is a real, non .values in repeated sampling

The rank of the data matrix **-06** X is equal to $(n\times k)$ where ,n is the number of observations in each variable, andk represents the number of columns (the number of independent variables), and the number of variablesK must be less than the number of observations so that the columns of the matrix are X vectors that are linearly independent of each other and there is no linear relationship .(Complete between independent variables (linearity

There is no relationship between the independent variables **-07** X_i and the random variable μ_i .

.The model has been accurately described and there are no description errors **-08**
.errors There are no data collection **-09**
.The ability to diagnose (distinguish) the relationship to be estimated **-10**

Third: Methods for estimating the parameters of the multiple linear model

In this estimation process, we are satisfied with only two explanatory variables, .same previous method used in the simple regression model as we follow the
) Ordinary Least Squares 01-03OLS) In light of the previously **: method** mentioned assumptions, the Ordinary Least Squares method can be used to :model, where estimate the parameters of the multiple linear
:Let us have the general multilinear model equation as follows

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \mu_i$$

Because of the statistical difficulty of estimating unknown population parameters, we draw a random sample and then estimate the parameters using :following equation the $Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_{i1} + \hat{\beta}_2 X_{i2} + e_i$

To estimate Y_i independently, this is done using the estimated regression :equation as follows $Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_{i1} + \hat{\beta}_2 X_{i2}$

The goal is to obtain values $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ that make the sum of the squares of the deviations as small as possible, i.e. reduce the value $\sum e_i^2$ to the lowest possible ,value $MIN \rightarrow \sum e_i^2$ i.e

We know from the above that

$$Y_i = Y_i + e_i \Rightarrow e_i = Y_i - Y_i$$

By squaring both sides of the equation and inserting the sum on both sides, we :find

$$\sum e_i^2 = \sum\left(Y_i - Y_i\right)^2 \Rightarrow \sum e_i^2 = \sum\left(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \hat{\beta}_2 X_{i2}\right)^2$$

Taking the partial derivatives of each $\left(\beta_0, \beta_1, \beta_2\right)$:we find

$$\frac{\partial \sum e_i^2}{\partial \beta_0} = -02 \sum\left(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \hat{\beta}_2 X_{i2}\right) = 0$$

) By dividing by-2 :and expanding the parentheses, we find (

$$\sum Y_i = n\hat{\beta}_0 - \hat{\beta}_1 \sum X_{i1} - \hat{\beta}_2 \sum X_{i2} \text{....................(01)}$$

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}_1} = -02\sum X_{i1}\left(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \hat{\beta}_2 X_{i2}\right) = 0$$

) By dividing by-2 :and expanding the parentheses, we find (

$$\sum X_{i1}Y_i = \hat{\beta}_0\sum X_{i1} - \hat{\beta}_1\sum X_{i1}^2 - \hat{\beta}_2\sum X_{i1}X_{i2} \ldots\ldots\ldots\ldots(02)$$

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}_2} = -02\sum X_{i2}\left(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \hat{\beta}_2 X_{i2}\right) = 0$$

) By dividing by-2 :and expanding the parentheses, we find (

$$\sum X_{i2}Y_i = \hat{\beta}_0\sum X_{i2} - \hat{\beta}_1\sum X_{i2}X_{i1} - \hat{\beta}_2\sum X_{i2}^2 \ldots\ldots\ldots\ldots(03)$$

Equations (01), (02), and (03) are called natural equations, which are used to estimate the three unknown parameters $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ and these equations can be ,

:solved in one of the following ways

previous three equations and after Based on the : **Determinant method 02-03** forming the matrix, the three unknown parameters can be found using Cramer's :method as follows

$$\sum Y_i = n\hat{\beta}_0 - \hat{\beta}_1\sum X_{i1} - \hat{\beta}_2\sum X_{i2} \ldots\ldots\ldots\ldots(01)$$

$$\sum X_{i1}Y_i = \hat{\beta}_0\sum X_{i1} - \hat{\beta}_1\sum X_{i1}^2 - \hat{\beta}_2\sum X_{i1}X_{i2} \ldots\ldots\ldots\ldots(02)$$

$$\sum X_{i2}Y_i = \hat{\beta}_0\sum X_{i2} - \hat{\beta}_1\sum X_{i2}X_{i1} - \hat{\beta}_2\sum X_{i2}^2 \ldots\ldots\ldots\ldots(03)$$

We form the matrix of the three equations and extract the values of the unknown Cramer's method as follows features using

$$\begin{bmatrix} \sum Y_i \\ \sum X_{i1}Y_i \\ \sum X_{i2}Y_i \end{bmatrix} = \begin{bmatrix} n & \sum X_{i1} & \sum X_{i2} \\ \sum X_{i1} & \sum X_{i1}^2 & \sum X_{i1}X_{i2} \\ \sum X_{i2} & \sum X_{i2}X_{i1} & \sum X_{i2}^2 \end{bmatrix} \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}$$

We now calculate the values of the unknown features $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$

$$\hat{\beta}_0 = \frac{(\sum Y_i)\times(D_{11}) - (\sum X_{i1}Y_i)\times(D_{12}) + (\sum X_{i2}Y_i)\times(D_{13})}{|D|}$$

$$\hat{\beta}_1 = \frac{-(\sum Y_i)\times(D_{21}) + (\sum X_{i1}Y_i)\times(D_{22}) - (\sum X_{i2}Y_i)\times(D_{23})}{|D|}$$

$$\hat{\beta}_2 = \frac{(\sum Y_i)\times(D_{31}) - (\sum X_{i1}Y_i)\times(D_{32}) + (\sum X_{i2}Y_i)\times(D_{33})}{|D|}$$

where D_{ij} are the least of the matrix while $|D|$ the general determinant of the is matrix given that

$$D_{11} = \left(\sum X_{i1}^2 \times \sum X_{i2}^2\right) - \left(\sum X_{i1} X_{i2}\right)^2$$

$$D_{12} = D_{21} = \left(\sum X_{i1} \times \sum X_{i2}^2\right) - \left(\sum X_{i1} X_{i2} \times \sum X_{i2}\right)$$

$$D_{13} = D_{31} = \left(\sum X_{i1} \times \sum X_{i2} X_{i1}\right) - \left(\sum X_{i1}^2 \times \sum X_{i2}\right)$$

$$D_{23} = D_{32} = \left(n \times \sum X_{i2} X_{i1}\right) - \left(\sum X_{i1} \times \sum X_{i2}\right)$$

$$D_{22} = \left(n \times \sum X_{i2}^2\right) - \left(\sum X_{i2}\right)^2$$

$$D_{33} = \left(n \times \sum X_{i1}^2\right) - \left(\sum X_{i1}\right)^2$$

To ensure that the parameters estimated **:Important note** $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ by the determinant method make the square of the residuals as small as possible, we order partial derivatives of the -calculate the determinant of the matrix of second) estimated parameters, that is, we extract the Hessian determinantH as (:follows

$$|H| = \begin{bmatrix} \frac{\partial^2 e_i^2}{\partial B_0^2} & \frac{\partial^2 e_i^2}{\partial B_0 \partial B_1} & \frac{\partial^2 e_i^2}{\partial B_0 \partial B_2} \\ \frac{\partial^2 e_i^2}{\partial B_1 \partial B_0} & \frac{\partial^2 e_i^2}{\partial B_1^2} & \frac{\partial^2 e_i^2}{\partial B_1 \partial B_2} \\ \frac{\partial^2 e_i^2}{\partial B_2 \partial B_0} & \frac{\partial^2 e_i^2}{\partial B_2 \partial B_1} & \frac{\partial^2 e_i^2}{\partial B_2^2} \end{bmatrix}$$

We perform the second derivation of the estimated parameters $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$

We know from the above that

For the teacher $\hat{\beta}_0$

$$\frac{\partial \sum e_i^2}{\partial \beta_0} = -02 \sum \left(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \hat{\beta}_2 X_{i2}\right) = 0$$

$$\frac{\partial \sum e_i^2}{\partial \beta_0} = -02 \sum Y_i + 02 n \beta_0 + 02 \beta_1 \sum X_{i1} + 02 \beta_2 \sum X_{i2} = 0$$

$$\frac{\partial^2 e_i^2}{\partial B_0^2} = 02n \quad , \frac{\partial^2 e_i^2}{\partial B_0 \partial B_1} = 02 \sum X_{i1} \quad , \frac{\partial^2 e_i^2}{\partial B_0 \partial B_2} = 2 \sum X_{i2}$$

For the teacher $\hat{\beta}_1$

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}_1} = -02\sum X_{i1}\left(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \hat{\beta}_2 X_{i2}\right) = 0$$

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}_1} = -02\sum X_{i1}Y_i + 02\beta_0\sum X_{i1} + 02\beta_1\sum X_{i1}^2 + 02\beta_2\sum X_{i1}X_{i2} = 0$$

$$\frac{\partial^2 e_i^2}{\partial B_1 \partial B_0} = 02\sum X_{i1},\ \frac{\partial^2 e_i^2}{\partial B_1^2} = 02\sum X_{i1}^2\ ,\frac{\partial^2 e_i^2}{\partial B_1 \partial B_2} = 02\sum X_{i1}X_{i2}$$

For the teacher $\hat{\beta}_2$

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}_2} = -02\sum X_{i2}\left(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \hat{\beta}_2 X_{i2}\right) = 0$$

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}_2} = -02\sum X_{i2}Y_i + 02\hat{\beta}_0\sum X_{i2} + 02\hat{\beta}_1\sum X_{i2}X_{i1} + 02\hat{\beta}_2\sum X_{i2}^2 = 0$$

$$\frac{\partial^2 e_i^2}{\partial B_2 \partial B_0} = 02\sum X_{i2}\ ,\frac{\partial^2 e_i^2}{\partial B_2 \partial B_1} = 02\sum X_{i2}X_{i1},\frac{\partial^2 e_i^2}{\partial B_2^2} = 02\sum X_{i2}^2$$

) We rewrite the second partial derivatives in the Hessian determinant matrixH (

$$|H| = \begin{bmatrix} \frac{\partial^2 e_i^2}{\partial B_0^2} & \frac{\partial^2 e_i^2}{\partial B_0 \partial B_1} & \frac{\partial^2 e_i^2}{\partial B_0 \partial B_2} \\ \frac{\partial^2 e_i^2}{\partial B_1 \partial B_0} & \frac{\partial^2 e_i^2}{\partial B_1^2} & \frac{\partial^2 e_i^2}{\partial B_1 \partial B_2} \\ \frac{\partial^2 e_i^2}{\partial B_2 \partial B_0} & \frac{\partial^2 e_i^2}{\partial B_2 \partial B_1} & \frac{\partial^2 e_i^2}{\partial B_2^2} \end{bmatrix} \Rightarrow |H| = \begin{bmatrix} 02n & 02\sum X_{i1} & 02\sum X_{i2} \\ 02\sum X_{i1} & 02\sum X_{i1}^2 & 02\sum X_{i1}X_{i2} \\ 02\sum X_{i2} & 02\sum X_{i2}X_{i1} & 02\sum X_{i2}^2 \end{bmatrix}$$

determinant must be greater than zero, i.e The Hessian

$$|H| = 02n\left[\left(04\sum X_{i1}^2 \times \sum X_{i2}^2\right) - 04\left(\sum X_{i2}X_{i1}\right)^{02}\right] - 02\sum X_{i1}\left[\left(04\sum X_{i1}\sum X_{i2}^2\right) - \left(04\sum X_{i1}X_{i2}\sum X_{i2}\right)\right]$$

$$+02\sum X_{i2}\left[\left(04\sum X_{i1}\sum X_{i2}X_{i1}\right) - \left(04\sum X_{i1}^2\sum X_{i2}\right)\right] > 0$$

,In this method, to find the values of unknown features **:Array method 03-03** $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$.the following rule is followed

$$B = \left(X^T X\right)^{-1} X^T Y$$

whereas

$$\left(X^T X\right) = \begin{bmatrix} n & \sum X_{i1} & \sum X_{i2} \\ \sum X_{i1} & \sum X_{i1}^2 & \sum X_{i1}X_{i2} \\ \sum X_{i2} & \sum X_{i2}X_{i1} & \sum X_{i2}^2 \end{bmatrix} \wedge \left(X^T Y\right) = \begin{bmatrix} \sum Y_i \\ \sum X_{i1}Y_i \\ \sum X_{i2}Y_i \end{bmatrix} \wedge B = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}$$

and that

$$\left(X^T X\right)^{-1} = \frac{1}{\left|\left(X^T X\right)\right|} \operatorname{adj}\left(X^T X\right)^t$$

$\left(X^T X\right)$ This matrix is called the FisherMatrix .

It is also possible to **:(Deviation method (estimating about the origin 04-03**

estimate $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ By deviations of the variables Y_i and X_{i1} and X_{i2} from their arithmetic mean $\overline{Y}$ and $\overline{X}_1$ and $\overline{X}_2$

We have the above

$$\sum Y_i = n\hat{\beta}_0 + \hat{\beta}_1 \sum X_{i1} + \hat{\beta}_2 \sum X_{i2} \text{..............................(01)}$$

$$\sum X_{i1} Y_i = \hat{\beta}_0 \sum X_{i1} + \hat{\beta}_1 \sum X_{i1}^2 + \hat{\beta}_2 \sum X_{i1} X_{i2} \text{..................(02)}$$

$$\sum X_{i2} Y_i = \hat{\beta}_0 \sum X_{i2} + \hat{\beta}_1 \sum X_{i2} X_{i1} + \hat{\beta}_2 \sum X_{i2}^2 \text{...............(03)}$$

And to create value $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ We follow the following steps

From Equation No. (01) we find

$$\sum Y_i = n\hat{\beta}_0 - \hat{\beta}_1 \sum X_{i1} - \hat{\beta}_2 \sum X_{i2} \Rightarrow \hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}_1 - \hat{\beta}_2 \overline{X}_2 \text{...............(04)}$$

We have the above

$$Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_{i1} + \hat{\beta}_2 X_{i2} + e_i \text{.........................(05)}$$

$$\overline{Y} = \hat{\beta}_0 + \hat{\beta}_1 \overline{X}_1 + \hat{\beta}_2 \overline{X}_2 \text{.......................................(06)}$$

:Subtracting (05) from (06) side by side, we find

$$Y_i - \overline{Y} = \left(\hat{\beta}_0 + \hat{\beta}_1 X_{i1} + \hat{\beta}_2 X_{i2} + e_i\right) - \left(\hat{\beta}_0 + \hat{\beta}_1 \overline{X}_1 + \hat{\beta}_2 \overline{X}_2\right)$$

$$Y_i - \overline{Y} = \hat{\beta}_1 \left(X_{i1} - \overline{X}_1\right) + \hat{\beta}_2 \left(X_{i2} - \overline{X}_2\right) + e_i$$

$$y_i = \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + e_i \text{..(07)}$$

By multiplying equation (07) by x_{i1} on both sides of the and entering the sum equation, we find

$$\sum x_{i1} y_i = \hat{\beta}_1 \sum x_{i1}^2 + \hat{\beta}_2 \sum x_{i1} x_{i2} + \sum e_i x_{i1}$$

Knowing that $\sum e_i x_{i1} = 0$ the equation becomes as follows

$$\sum x_{i1} y_i = \beta_1 \sum x_{i1}^2 + \beta_2 \sum x_{i1} x_{i2} \text{.......................(08)}$$

By multiplying equation (07) by x_{i2} of the and entering the sum on both sides equation, we find

$$\sum x_{i2} y_i = \beta_1 \sum x_{i1} x_{i2} + \beta_2 \sum x_{i2}^2 + \sum e_i x_{i2}$$

Knowing that $\sum e_i x_{i2} = 0$ the equation becomes as follows

$$\sum x_{i2}y_i = \beta_1 \sum x_{i1}x_{i2} + \beta_2 \sum x_{i2}^2 \text{..........................(09)}$$

Equations (08) and (09) are called natural equations and can be solved either .method or the matrix method using the determinant

.(We have the previous equations (08) and (09 **:Determinant method 01-03-04**

$$\sum x_{i1}y_i = \beta_1 \sum x_{i1}^2 + \beta_2 \sum x_{i1}x_{i2} \text{.......................(08)}$$

$$\sum x_{i2}y_i = \beta_1 \sum x_{i1}x_{i2} + \beta_2 \sum x_{i2}^2 \text{.......................(09)}$$

:We form the matrix system of the two equations as follows

$$\begin{bmatrix} \sum x_{i1}y_i \\ \sum x_{i2}y_i \end{bmatrix} = \begin{bmatrix} \sum x_{i1}^2 & \sum x_{i1}x_{i2} \\ \sum x_{i1}x_{i2} & \sum x_{i2}^2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

:general determinant of the matrix as follows We calculate the

$$|D| = \left(\sum x_{i1}^2 \times \sum x_{i2}^2\right) - \left(\sum x_{i1}x_{i2}\right)$$

We extract the values of the unknown features $\left(\hat{\beta}_1, \hat{\beta}_2\right)$ using Cramer's method as :follows

$$\beta_1 = \frac{\left(\sum x_{i1}y_i\right)\left(\sum x_{i2}^2\right) - \left(\sum x_{i2}y_i\right)\left(\sum x_{i1}x_{i2}\right)}{\left(\sum x_{i1}^2 \times \sum x_{i2}^2\right) - \left(\sum x_{i1}x_{i2}\right)}$$

$$\beta_2 = \frac{\left(\sum x_{i2}y_i\right)\left(\sum x_{i1}^2\right) - \left(\sum x_{i1}y_i\right)\left(\sum x_{i1}x_{i2}\right)}{\left(\sum x_{i1}^2 \times \sum x_{i2}^2\right) - \left(\sum x_{i1}x_{i2}\right)}$$

We know from the above that the basic rule in **:The Matrix Method 02-04-03** solving the matrix method is

$$B = \left(X^T X\right)^{-1} X^T Y$$

whereas $\left(X^T X\right) = \begin{bmatrix} \sum x_{i1}^2 & \sum x_{i1}x_{i2} \\ \sum x_{i1}x_{i2} & \sum x_{i2}^2 \end{bmatrix} \wedge \left(X^T Y\right) = \begin{bmatrix} \sum x_{i1}y_i \\ \sum x_{i2}y_i \end{bmatrix} \wedge \left(B\right) = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$

Knowing that $\left(X^T X\right)^{-1} = \frac{1}{\left|\left(X^T X\right)\right|} \operatorname{adj}\left(X^T X\right)^t$

following table Let you have the data shown in the : **Example**

116	20	18	16	14	12	10	09	07	06	04	Y_i
95	17	15	14	12	10	09	07	05	04	02	X_{i1}
98	15	14	13	11	09	08	08	10	07	03	X_{i2}

equation using the following Find the estimated multiple regression **:Required** methods

Determinants method-01

Chapter three : Multiple Linear Regression

Matrix method-02
.Deviations method-03
We know that the estimated multiple linear regression equation is **:Solution** given by the following relationship

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2}$$

We know from the above that we have three **:The method of determinants-01** :natural equations

$$\sum Y_i = n\hat{\beta}_0 + \hat{\beta}_1 \sum X_{i1} + \hat{\beta}_2 \sum X_{i2} \ldots\ldots\ldots\ldots(01)$$

$$\sum X_{i1} Y_i = \hat{\beta}_0 \sum X_{i1} + \hat{\beta}_1 \sum X_{i1}^2 + \hat{\beta}_2 \sum X_{i1} X_{i2} \ldots\ldots\ldots\ldots(02)$$

$$\sum X_{i2} Y_i = \hat{\beta}_0 \sum X_{i2} + \hat{\beta}_1 \sum X_{i2} X_{i1} + \hat{\beta}_2 \sum X_{i2}^2 \ldots\ldots\ldots\ldots(03)$$

We calculate the following values from the previous table

$$n = 10, \sum Y_i = 116, \sum X_{i1} = 95, \sum X_{i2} = 98, \sum X_{i1} Y_i = 1342, \sum X_{i2} Y_i = 1298, \sum X_{i1}^2 = 1129, \sum X_{i2}^2 = 1078$$
$$\sum X_{i1} X_{i2} = 1081, \sum Y_i^2 = 1602$$

:find Substituting in the previous three equations we

$$\begin{cases} \sum Y_i = n\hat{\beta}_0 + \hat{\beta}_1 \sum X_{i1} + \hat{\beta}_2 \sum X_{i2} \\ \sum X_{i1} Y_i = \hat{\beta}_0 \sum X_{i1} + \hat{\beta}_1 \sum X_{i1}^2 + \hat{\beta}_2 \sum X_{i1} X_{i2} \\ \sum X_{i2} Y_i = \hat{\beta}_0 \sum X_{i2} + \hat{\beta}_1 \sum X_{i2} X_{i1} + \hat{\beta}_2 \sum X_{i2}^2 \end{cases} \Rightarrow \begin{cases} 116 = 10\hat{\beta}_0 + 95\hat{\beta}_1 + 98\hat{\beta}_2 \\ 1342 = 95\hat{\beta}_0 + 1129\hat{\beta}_1 + 1081\hat{\beta}_2 \\ 1298 = 98\hat{\beta}_0 + 1081\hat{\beta}_1 + 1078\hat{\beta}_2 \end{cases}$$

We form a matrix to extract the values of the unknown features $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ as :follows

$$\begin{bmatrix} 116 \\ 1342 \\ 1298 \end{bmatrix} = \begin{bmatrix} 10 & 95 & 98 \\ 95 & 1129 & 1081 \\ 98 & 1081 & 1078 \end{bmatrix} \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}$$

We calculate the general determinant of the matrix as follows

$$|D| = 10 \times \left[(1129 \times 1078) - (1081)^2\right] - 95\left[(95 \times 1078) - (1081 \times 98)\right] + 98\left[(95 \times 1081) - (1129 \times 98)\right]$$

$$|D| = 41364$$

We extract the values of the unknown features $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ using Cramer's method, and before that, we extract the minima of the matrix

$D_{11} = (1129\times1078) - (1081)^2 = 48501$

$D_{12} = D_{21} = (95\times1078) - (1081\times98) = 3528$

$D_{13} = D_{31} = (95\times1081) - (1129\times98) = -7947$

$D_{23} = D_{32} = (10\times1081) - (95\times98) = -1500$

$D_{22} = (10\times1078) - (98)^2 = 1176$

$D_{33} = (10\times1129) - (95)^2 = 2265$

So the values of the unknown features $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ are as follows

$$\hat{\beta}_0 = \frac{(116)\times(48501) + (1342)\times(3528) + (1298)\times(-7947)}{41364} = 01,09965$$

$$\hat{\beta}_1 = \frac{(116)\times(3528) + (1342)\times(1176) + (1298)\times(-1500)}{41364} = 0.97766$$

$$\hat{\beta}_2 = \frac{(116)\times(-7947) + (1342)\times(-1500) + (1298)\times(2265)}{41364} = 0,12373$$

:So the estimated multiple linear regression equation is

$$Y_i = 01,09965 + 0,97766X_{i1} + 0,12373X_{i2}$$

Calculating the second partial derivatives and making sure that the parameters - estimated $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ the square of the by the determinant method make remainders as small as possible, we extract the Hessian determinant

$$|H| = \begin{bmatrix} \frac{\partial^2 e_i^2}{\partial B_0^2} & \frac{\partial^2 e_i^2}{\partial B_0 \partial B_1} & \frac{\partial^2 e_i^2}{\partial B_0 \partial B_2} \\ \frac{\partial^2 e_i^2}{\partial B_1 \partial B_0} & \frac{\partial^2 e_i^2}{\partial B_1^2} & \frac{\partial^2 e_i^2}{\partial B_1 \partial B_2} \\ \frac{\partial^2 e_i^2}{\partial B_2 \partial B_0} & \frac{\partial^2 e_i^2}{\partial B_2 \partial B_1} & \frac{\partial^2 e_i^2}{\partial B_2^2} \end{bmatrix} \Rightarrow |H| = \begin{bmatrix} 20 & 190 & 196 \\ 190 & 2258 & 2162 \\ 196 & 2162 & 2156 \end{bmatrix}$$

$$|H| = 20\times\left[(2258\times2156) - (2162)^2\right] - 190\left[(190\times2156) - (2162\times196)\right]$$

$$+196\left[(2162\times190) - (2258\times196)\right]$$

$$|H| = 330912 > 0$$

So the Hessian determinant is greater than zero, which means that the square of .the remainders is as small as possible

,ind the values of unknown featuresTo f **:Matrix method-02** $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ the following rule is followed

$$B = \left(X^T X\right)^{-1}\left(X^T Y\right)$$

,To find the inverse of the matrix $\left(X^T X\right)^{-1}$ we first extract $\left(X^T X\right)$

We know from the above that

$$\left(X^TX\right)=\begin{bmatrix} n & \sum X_{i1} & \sum X_{i2} \\ \sum X_{i1} & \sum X_{i1}^2 & \sum X_{i1}X_{i2} \\ \sum X_{i2} & \sum X_{i2}X_{i1} & \sum X_{i2}^2 \end{bmatrix} \Rightarrow \left(X^TX\right)=\begin{bmatrix} 10 & 95 & 98 \\ 95 & 1129 & 1081 \\ 98 & 1081 & 1078 \end{bmatrix}$$

and that $\left(X^TX\right)^{-1}=\frac{1}{\left|\left(X^TX\right)\right|}\text{adj}\left(X^TX\right)^t$

We first extract the matrix of conjugates (minimums) as follows

$$\left(X^TX\right)^C=\begin{bmatrix} 48501 & 3528 & -7947 \\ 3528 & 1176 & -1500 \\ -7947 & -1500 & 2265 \end{bmatrix}$$

A matrix converter $\left(X^TX\right)^T$ is a matrix converter $\left(X^TX\right)^C$

permission

$$\left(X^TX\right)^{-1}=\frac{1}{\left|\left(X^TX\right)\right|}\text{adj}\left(X^TX\right)^t \Rightarrow \left(X^TX\right)^{-1}=\frac{1}{41364}\begin{bmatrix} 48501 & 3528 & -7947 \\ 3528 & 1176 & -1500 \\ -7947 & -1500 & 2265 \end{bmatrix}$$

We now calculate the value $\left(X^TY\right)$

$$\left(X^TY\right)=\begin{bmatrix} \sum Y_i \\ \sum X_{i1}Y_i \\ \sum X_{i2}Y_i \end{bmatrix} \Rightarrow \left(X^TY\right)=\begin{bmatrix} 116 \\ 1342 \\ 1298 \end{bmatrix}$$

Therefore, the values of the unknown features $\left(\hat{\beta}_0,\hat{\beta}_1,\hat{\beta}_2\right)$ are

$$B=\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}=\left(X^TX\right)^{-1}\left(X^TY\right) \Rightarrow B=\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}=\frac{1}{41364}\begin{bmatrix} 48501 & 3528 & -7947 \\ 3528 & 1176 & -1500 \\ -7947 & -1500 & 2265 \end{bmatrix}\times\begin{bmatrix} 116 \\ 1342 \\ 1298 \end{bmatrix}=\begin{bmatrix} 01,09965 \\ 0,97766 \\ 0,12373 \end{bmatrix}$$

:regression equation is So the estimated multiple linear

$$Y_i = 01{,}09965 + 0{,}97766X_{i1} + 0{,}12373X_{i2}$$

We have the above **:The method of deviations-03**

$$\hat{\beta}_0=\overline{Y}-\hat{\beta}_1\overline{X}_1-\hat{\beta}_2\overline{X}_2$$

$$\sum x_{i1}y_i=\beta_1\sum x_{i1}^2+\beta_2\sum x_{i1}x_{i2}$$

$$\sum x_{i2}y_i=\beta_1\sum x_{i1}x_{i2}+\beta_2\sum x_{i2}^2$$

The values of unknown features can be found according to the following two :methods natural equations we form the From the two **:Method of determinants 01-03** matrix system

$$\sum x_{i1}y_i = \beta_1 \sum x_{i1}^2 + \beta_2 \sum x_{i1}x_{i2}$$

$$\sum x_{i2}y_i = \beta_1 \sum x_{i1}x_{i2} + \beta_2 \sum x_{i2}^2$$

$$\begin{bmatrix} \sum x_{i1}y_i \\ \sum x_{i2}y_i \end{bmatrix} = \begin{bmatrix} \sum x_{i1}^2 & \sum x_{i1}x_{i2} \\ \sum x_{i1}x_{i2} & \sum x_{i2}^2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

:We first extract the following values

$$\overline{X}_1 = 09,50, \overline{X}_2 = 09,80, \overline{Y} = 11,60$$

$$\sum y_i^2 = \sum Y_i^2 - n \times \overline{Y}^2 \Rightarrow \sum y_i^2 = 1602 - (10)(11,60)^2 = 256,40$$

$$\sum x_{i1}y_i = \sum X_{i1}Y_i - n\overline{X}_1\overline{Y} \Rightarrow \sum x_{i1}y_i = 1342 - (10)(09,50)(11,60) = 240$$

$$\sum x_{i2}y_i = \sum X_{i2}Y_i - n\overline{X}_2\overline{Y} \Rightarrow \sum x_{i2}y_i = 1298 - (10)(09,80)(11,60) = 161,20$$

$$\sum x_{i1}x_{i2} = \sum X_{i1}X_{i2} - n\overline{X}_1\overline{X}_2 \Rightarrow \sum x_{i1}x_{i2} = 1081 - (10)(09,50)(09,80) = 150$$

$$\sum x_{i1}^2 = \sum X_{i1}^2 - n\overline{X}_1^2 \Rightarrow \sum x_{i1}^2 = 1129 - (10)(09.50)^2 = 226,50$$

$$\sum x_{i2}^2 = \sum X_{i2}^2 - n\overline{X}_2^2 \Rightarrow \sum x_{i2}^2 = 1078 - (10)(09.80)^2 = 117,60$$

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$$\begin{bmatrix} \sum x_{i1}y_i \\ \sum x_{i2}y_i \end{bmatrix} = \begin{bmatrix} \sum x_{i1}^2 & \sum x_{i1}x_{i2} \\ \sum x_{i1}x_{i2} & \sum x_{i2}^2 \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 240 \\ 161,20 \end{bmatrix} = \begin{bmatrix} 226,50 & 150 \\ 150 & 117,60 \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}$$

We calculate the general determinant of the matrix

$$\begin{bmatrix} 226,50 & 150 \\ 150 & 117,60 \end{bmatrix} \Rightarrow |D| = (226,50 \times 117,60) - (150)^2 = 4136,40$$

We extract the values of the unknown features for each $(\hat{\beta}_1, \hat{\beta}_2)$ using Cramer's method

$$\hat{\beta}_1 = \frac{(240)(117,60) - (161,20)(150)}{4136,40} = 0.97766$$

$$\hat{\beta}_2 = \frac{(161,20)(226.50) - (240)(150)}{4136,40} = 0,12373$$

Substituting a value $(\hat{\beta}_1, \hat{\beta}_2)$:into the first equation, we find

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1\overline{X}_1 - \hat{\beta}_2\overline{X}_2 \Rightarrow \hat{\beta}_0 = 11,60 - (0,97766)(09,50) - (0,12373)(09,80) = 01,09967$$

:So the estimated multiple linear regression equation is

$$Y_i = 01,09965 + 0,97766X_{i1} + 0,12373X_{i2}$$

We have the above **:Array Method 02-03** $B = (X^T X)^{-1} X^T Y$

We extract the value of the matrix $\left(X^{T}X\right)$ there we calculate the and from inverse of the matrix $\left(X^{T}X\right)^{-1}$

$$\left(X^{T}X\right)=\begin{bmatrix}\sum x_{i1}^{2} & \sum x_{i1}x_{i2}\\ \sum x_{i1}x_{i2} & \sum x_{i2}^{2}\end{bmatrix}\Rightarrow\left(X^{T}X\right)=\begin{bmatrix}226,50 & 150\\ 150 & 117,60\end{bmatrix}$$

and that $\left(X^{T}X\right)^{-1}=\dfrac{1}{\left|\left(X^{T}X\right)\right|}\operatorname{adj}\left(X^{T}X\right)^{t}$

We first extract the matrix of conjugates (minimums) as follows

$$\left(X^{T}X\right)^{C}=\begin{bmatrix}117,60 & -150\\ -150 & 226,50\end{bmatrix}$$

A matrix converter $\left(X^{T}X\right)^{T}$ is a matrix converter $\left(X^{T}X\right)^{C}$ permission

$$\left(X^{T}X\right)^{-1}=\frac{1}{\left|\left(X^{T}X\right)\right|}\operatorname{adj}\left(X^{T}X\right)^{t}\Rightarrow\left(X^{T}X\right)^{-1}=\frac{1}{4136,4}\begin{bmatrix}117,60 & -150\\ -150 & 226,50\end{bmatrix}$$

We now calculate the value $\left(X^{T}Y\right)$

$$\left(X^{T}Y\right)=\begin{bmatrix}\sum x_{i1}y_{i}\\ \sum x_{i2}y_{i}\end{bmatrix}\Rightarrow\left(X^{T}Y\right)=\begin{bmatrix}240\\ 161.20\end{bmatrix}$$

Therefore, the values of the unknown features $\left(\hat{\beta}_1,\hat{\beta}_2\right)$ are

$$B=\begin{bmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{bmatrix}=\left(X^{T}X\right)^{-1}\left(X^{T}Y\right)\Rightarrow B=\begin{bmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{bmatrix}=\frac{1}{4136,4}\begin{bmatrix}117,60 & -150\\ -150 & 226,50\end{bmatrix}\times\begin{bmatrix}240\\ 161.20\end{bmatrix}=\begin{bmatrix}0,97766\\ 0,12373\end{bmatrix}$$

Substituting a value $\left(\hat{\beta}_1,\hat{\beta}_2\right)$:into the first equation, we find

$$\hat{\beta}_0=\overline{Y}-\hat{\beta}_1\overline{X}_1-\hat{\beta}_2\overline{X}_2\Rightarrow\hat{\beta}_0=11,60-(0,97766)(09,50)-(0,12373)(09,80)=01,09967$$

:equation is So the estimated multiple linear regression

$$Y_i=01,09965+0,97766X_{i1}+0,12373X_{i2}$$

:(Fourth: Variance and covariance (covariance

The variance and covariance matrix of the multiple linear regression model :estimators can be found as follows

We have it above

$$B=\left(X^{T}X\right)^{-1}\left(X^{T}Y\right)\ldots\ldots\ldots\ldots\ldots\ldots(01)$$

$$Y=BX+\mu_i\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(02)$$

:into (01) we find (02) Substituting

$$B = \left(X^T X\right)^{-1} X^T \left(BX + \mu_i\right) = \left(X^T X\right)^{-1} \left(X^T X\right) B + \left(X^T X\right)^{-1} X^T \mu_i$$

$$= B + \left(X^T X\right)^{-1} X^T \mu_i \text{......................} (03)$$

By inserting the mathematical expectation on both sides of equation (03), we :find $E\left(B\right) = E(B) + \left(X^T X\right)^{-1} X^T E(\mu_i)$

Knowing that we have the assumptions of the multiple linear regression model that $E\left(\mu_i\right) = 0$

,Therefore $E\left(B\right) = B$

So it is an unbiased estimate of the real value B $\left(B\right)$

The formula for calculating the values of both the variance of these features and them can be reached as follows the covariance between

(We have from equation (03

$$B = B + \left(X^T X\right)^{-1} X^T \mu_i \Rightarrow B - B = \left(X^T X\right)^{-1} X^T \mu_i \text{........} (04)$$

:The vectorized variance and covariance matrix can be written as $\left(B\right)$

$$\operatorname{var}\left(\hat{B}\right) - \operatorname{cov}\left(B\right) = E\left[\left(\hat{B} - B\right)\left(\hat{B} - B\right)^T\right]$$

Referring to Equation No. (04), we find

$$\operatorname{var}\left(B\right) - \operatorname{cov}\left(B\right) = E\left[\left(B - B\right)\left(B - B\right)^T\right] \Rightarrow \operatorname{var}\left(B\right) - \operatorname{cov}\left(B\right) = E\left[\left(\left(X^T X\right)^{-1} X^T \mu_i\right)\left(\left(X^T X\right)^{-1} X^T \mu_i\right)^T\right]$$

$$\Rightarrow \operatorname{var}\left(B\right) - \operatorname{cov}\left(B\right) = E\left[\left(\left(X^T X\right)^{-1} X^T \mu_i\right)\left(\left(X^T X\right)^{-1} X \mu_i^T\right)\right]$$

$$\Rightarrow \operatorname{var}\left(B\right) - \operatorname{cov}\left(B\right) = E\left[\left(X^T X\right)^{-1} \left(X^T X\right)\left(X^T X\right)^{-1} \mu_i \mu_i^T\right]$$

$$\Rightarrow \operatorname{var}\left(B\right) - \operatorname{cov}\left(B\right) = \left(X^T X\right)^{-1} E\left(\mu_i \mu_i^T\right)$$

$$\Rightarrow \operatorname{var}\left(B\right) - \operatorname{cov}\left(B\right) = \sigma_\mu^2 \times \left(X^T X\right)^{-1}$$

:So the formula for variance and covariance is

$$\operatorname{var}\left(B\right) - \operatorname{cov}\left(B\right) = \sigma_\mu^2 \times \left(X^T X\right)^{-1}$$

It is clear from the last relationship that the value of the variance of any element B is the product of multiplying a value σ_μ^2 by the corresponding elements located on the diagonal of a matrix $\left(X^T X\right)^{-1}$ just as the value of the common , variance between any two elements B is the product of the product σ_μ^2 of the corresponding element located outside the diagonal of the matrix $\left(X^T X\right)^{-1}$.

If the deviation method is used, the matrix **:Note 01** $\left(X^{T}X\right)^{-1}$ measured by deviations does not include the variance of the fixed term of the model $var\left(\hat{\beta}_0\right)$, nor does it include the covariance of the fixed term with any marginal slope. In other words, it does not include $cov\left(\hat{\beta}_0,\hat{\beta}_j\right)$ j=1,2,3,........,k d therefore it is an , necessary to derive a special formula for them. If we have a model with only two variables, we can Finding this relationship and generalizing it to more than two independent variables

Estimating the constant term of variance $var\left(\hat{\beta}_0\right)$ The constant term of a **:** model with two independent variables can be estimated as follows

We know from the above that $\hat{\beta}_0=\overline{Y}-\hat{\beta}_1\overline{X}_1-\hat{\beta}_2\overline{X}_2$..........(01)

:Note that the general model for multiple linear regression is

$Y_i=\beta_0+\beta_1X_{i1}+\beta_2X_{i2}+\mu_i$.........(02)

And its average is $\overline{Y}=\beta_0+\beta_1\overline{X}_1+\beta_2\overline{X}_2+\overline{\mu}$.........(03)

:Substituting (03) into (01) we find

$$\hat{\beta}_0=\beta_0+\beta_1\overline{X}_1+\beta_2\overline{X}_2+\overline{\mu}-\hat{\beta}_1\overline{X}_1-\hat{\beta}_2\overline{X}_2\Rightarrow$$

$$\hat{\beta}_0-\beta_0=\left[-\overline{X}_1\left(\hat{\beta}_1-\beta_1\right)-\overline{X}_2\left(\hat{\beta}_2-\beta_2\right)\right]+\left[\overline{\mu}\right]\text{....................(04)}$$

By squaring both sides of the equation and introducing the mathematical :we find ,expectation on both sides of the final equation

$$E\left(\hat{\beta}_0-\beta_0\right)^2=\overline{X}_1^2E\left(\hat{\beta}_1-\beta_1\right)^2+02\overline{X}_1\overline{X}_2E\left(\hat{\beta}_1-\beta_1\right)\left(\hat{\beta}_2-\beta_2\right)^2+\overline{X}_2^2E\left(\hat{\beta}_2-\beta_2\right)^2+E\left(\overline{\mu}\right)^2$$

$$E\left(\hat{\beta}_0-\beta_0\right)^2=\overline{X}_1^2\,var\left(\hat{\beta}_1\right)+02\overline{X}_1\overline{X}_2\,cov\left(\hat{\beta}_1,\hat{\beta}_2\right)+\overline{X}_2^2\,var\left(\hat{\beta}_2\right)+\frac{\sigma_\mu^2}{n}$$

$$E\left(\hat{\beta}_0-\beta_0\right)^2=\left[\overline{X}_1\quad\overline{X}_2\right]\begin{bmatrix}var\left(\hat{\beta}_1\right) & cov\left(\hat{\beta}_1,\hat{\beta}_2\right)\\ cov\left(\hat{\beta}_2,\hat{\beta}_1\right) & var\left(\hat{\beta}_2\right)\end{bmatrix}\begin{bmatrix}\overline{X}_1\\ \overline{X}_2\end{bmatrix}+\frac{\sigma_\mu^2}{n}$$

The last equation can be written in the following form

$$var\left(\hat{\beta}_0\right)=E\left(\hat{\beta}_0-\beta_0\right)^2=\overline{X}^T\times\sigma_\mu^2\times\left(X^TX\right)^{-1}\times\overline{X}+\frac{\sigma_\mu^2}{n}$$

It is clear from the last relationship that the matrix $\left(X^TX\right)^{-1}$ was calculated independent variables are measured by around the mean point, meaning that all deviations and not by original values. As for the vector $\overline{X}$ It represents the .averages of the independent variables

Estimating the covariance of the fixed term $var\left(\hat{\beta}_0\right)$ The covariance between **:** the estimated constant term $\left(\hat{\beta}_0\right)$ and any estimated marginal slope for this model that includes two independent variables can be reached through the following .relationship

.(No. (04 We have from the previous relationship

$$\hat{\beta}_0-\beta_0=\left[-\overline{X}_1\left(\hat{\beta}_1-\beta_1\right)-\overline{X}_2\left(\hat{\beta}_2-\beta_2\right)\right]+\left[\overline{\mu}\right]$$

The covariance between the constant term and the slopes of the first and second terms can be obtained

The common variance between the constant term-$\left(\beta_0\right)$ and the first marginal slope$\left(\hat{\beta}_1\right)$ The common variance is given according to the following **:** relationship

$$\hat{\beta}_0-\beta_0=\left[-\overline{X}_1\left(\hat{\beta}_1-\beta_1\right)-\overline{X}_2\left(\hat{\beta}_2-\beta_2\right)\right]+\left[\overline{\mu}\right]\Rightarrow\left(\hat{\beta}_0-\beta_0\right)+\overline{X}_1\left(\hat{\beta}_1-\beta_1\right)=-\overline{X}_2\left(\hat{\beta}_2-\beta_2\right)+\left[\overline{\mu}\right]$$

:Digitizing both sides of the equation and introducing the expectation, we find

$$E\left[\left(\hat{\beta}_0-\beta_0\right)+\overline{X}_1\left(\hat{\beta}_1-\beta_1\right)\right]^2=E\left[-\overline{X}_2\left(\hat{\beta}_2-\beta_2\right)+\left(\overline{\mu}\right)\right]^2$$

$$E\left[\left(\hat{\beta}_0-\beta_0\right)^2+\overline{X}_1^2\left(\hat{\beta}_1-\beta_1\right)^2+02\overline{X}_1\left(\hat{\beta}_0-\beta_0\right)\left(\hat{\beta}_1-\beta_1\right)\right]=E\left[\overline{X}_2^2\left(\hat{\beta}_2-\beta_2\right)^2+\left(\overline{\mu}\right)^2-02\left(\hat{\beta}_2-\beta_2\right)\left(\overline{\mu}\right)\right]$$

$$E\left(\hat{\beta}_0-\beta_0\right)^2+\overline{X}_1^2E\left(\hat{\beta}_1-\beta_1\right)^2+02\overline{X}_1E\left(\hat{\beta}_0-\beta_0\right)\left(\hat{\beta}_1-\beta_1\right)=\overline{X}_2^2E\left(\hat{\beta}_2-\beta_2\right)^2+E\left(\overline{\mu}\right)^2-02E\left(\hat{\beta}_2-\beta_2\right)\left(\overline{\mu}\right)$$

$$\operatorname{var}\left(\hat{\beta}_0\right)+\overline{X}_1^2\operatorname{var}\left(\hat{\beta}_1\right)+02\overline{X}_1\operatorname{cov}\left(\widehat{\beta_0,\beta_1}\right)=\overline{X}_2^2\operatorname{var}\left(\hat{\beta}_2\right)+\operatorname{var}\left(\overline{\mu}\right)-02\operatorname{cov}\left(\hat{\beta}_2,\overline{\mu}\right)$$

$$02\overline{X}_1\operatorname{cov}\left(\widehat{\beta_0,\beta_1}\right)=-\operatorname{var}\left(\hat{\beta}_0\right)-\overline{X}_1^2\operatorname{var}\left(\hat{\beta}_1\right)+\overline{X}_2^2\operatorname{var}\left(\hat{\beta}_2\right)+\operatorname{var}\left(\overline{\mu}\right)$$

And substitute the value $\operatorname{var}\left(\hat{\beta}_0\right)$ into the last relation we find

$$02\overline{X}_1\operatorname{cov}\left(\beta_0,\beta_1\right)=-\operatorname{var}\left(\hat{\beta}_0\right)-\overline{X}_1^2\operatorname{var}\left(\hat{\beta}_1\right)+\overline{X}_2^2\operatorname{var}\left(\hat{\beta}_2\right)+\operatorname{var}\left(\overline{\mu}\right)$$

We calculate the left side of the equation after substituting a value $\operatorname{var}\left(\hat{\beta}_0\right)$

$$-\overline{X}_1^2\operatorname{var}\left(\hat{\beta}_1\right)-02\overline{X}_1\overline{X}_2\operatorname{cov}\left(\hat{\beta}_1,\hat{\beta}_2\right)-\cancel{\overline{X}_2^2\operatorname{var}\left(\hat{\beta}_2\right)}-\cancel{\frac{\sigma_\mu^2}{n}}-\overline{X}_1^2\operatorname{var}\left(\hat{\beta}_1\right)+\cancel{\overline{X}_2^2\operatorname{var}\left(\hat{\beta}_2\right)}+\frac{\sigma_\mu^2}{n}$$

$$-02\overline{X}_1^2\operatorname{var}\left(\hat{\beta}_1\right)-02\overline{X}_1\overline{X}_2\operatorname{cov}\left(\hat{\beta}_1,\hat{\beta}_2\right)$$

permission $02\overline{X}_1\operatorname{cov}\left(\beta_0,\beta_1\right)=-02\overline{X}_1^2\operatorname{var}\left(\hat{\beta}_1\right)-02\overline{X}_1\overline{X}_2\operatorname{cov}\left(\hat{\beta}_1,\hat{\beta}_2\right)$

$$\overline{X}_1\operatorname{cov}\left(\beta_0,\beta_1\right)=-\overline{X}_1^2\operatorname{var}\left(\hat{\beta}_1\right)-\overline{X}_1\overline{X}_2\operatorname{cov}\left(\hat{\beta}_1,\hat{\beta}_2\right)\Rightarrow$$

$$\operatorname{cov}\left(\beta_0,\beta_1\right)=\frac{-\overline{X}_1^2\operatorname{var}\left(\hat{\beta}_1\right)-\overline{X}_1\overline{X}_2\operatorname{cov}\left(\hat{\beta}_1,\hat{\beta}_2\right)}{\overline{X}_1}$$

The common variance between the constant term-$\left(\beta_0\right)$ and the first marginal slope $\left(\hat{\beta}_2\right)$ The common variance is given according to the following **:** relationship

$$\hat{\beta}_0-\beta_0=\left[-\bar{X}_1\left(\hat{\beta}_1-\beta_1\right)-\bar{X}_2\left(\hat{\beta}_2-\beta_2\right)\right]+\left[\bar{\mu}\right]\Rightarrow\left(\hat{\beta}_0-\beta_0\right)+\bar{X}_2\left(\hat{\beta}_2-\beta_2\right)=-\bar{X}_1\left(\hat{\beta}_1-\beta_1\right)+\left[\bar{\mu}\right]$$

:Digitizing both sides of the equation and introducing the expectation, we find

$$E\left[\left(\hat{\beta}_0-\beta_0\right)+\bar{X}_2\left(\hat{\beta}_2-\beta_2\right)\right]^2=E\left[-\bar{X}_1\left(\hat{\beta}_1-\beta_1\right)+\left(\bar{\mu}\right)\right]^2$$

$$E\left[\left(\hat{\beta}_0-\beta_0\right)^2+\bar{X}_2^2\left(\hat{\beta}_2-\beta_2\right)^2+02\bar{X}_2\left(\hat{\beta}_0-\beta_0\right)\left(\hat{\beta}_2-\beta_2\right)\right]=E\left[\bar{X}_1^2\left(\hat{\beta}_1-\beta_1\right)^2+\left(\bar{\mu}\right)^2-02\left(\hat{\beta}_1-\beta_1\right)\left(\bar{\mu}\right)\right]$$

$$E\left(\hat{\beta}_0-\beta_0\right)^2+\bar{X}_2^2E\left(\hat{\beta}_2-\beta_2\right)^2+02\bar{X}_2E\left(\hat{\beta}_0-\beta_0\right)\left(\hat{\beta}_2-\beta_2\right)=\bar{X}_1^2E\left(\hat{\beta}_1-\beta_1\right)^2+E\left(\bar{\mu}\right)^2-02E\left(\hat{\beta}_1-\beta_1\right)\left(\bar{\mu}\right)$$

$$\text{var}\left(\hat{\beta}_0\right)+\bar{X}_2^2\,\text{var}\left(\hat{\beta}_2\right)+02\bar{X}_2\,\text{cov}\left(\widehat{\beta_0,\beta_2}\right)=\bar{X}_1^2\,\text{var}\left(\hat{\beta}_1\right)+\text{var}\left(\bar{\mu}\right)-02\,\text{cov}\left(\hat{\beta}_1,\bar{\mu}\right)$$

$$02\bar{X}_2\,\text{cov}\left(\widehat{\beta_0,\beta_2}\right)=-\text{var}\left(\hat{\beta}_0\right)-\bar{X}_2^2\,\text{var}\left(\hat{\beta}_2\right)+\bar{X}_1^2\,\text{var}\left(\hat{\beta}_1\right)+\text{var}\left(\bar{\mu}\right)$$

And substitute the value $\text{var}\left(\hat{\beta}_0\right)$ into the last relation we find

$$02\bar{X}_2\,\text{cov}\left(\beta_0,\beta_2\right)=-\text{var}\left(\hat{\beta}_0\right)-\bar{X}_2^2\,\text{var}\left(\hat{\beta}_2\right)+\bar{X}_1^2\,\text{var}\left(\hat{\beta}_1\right)+\text{var}\left(\bar{\mu}\right)$$

left side of the equation after substituting a value We calculate the $\text{var}\left(\hat{\beta}_0\right)$

$$-\bar{X}_1^2\,\text{var}\left(\hat{\beta}_1\right)-02\bar{X}_2\bar{X}_1\,\text{cov}\left(\hat{\beta}_2,\hat{\beta}_1\right)-\bar{X}_2^2\,\text{var}\left(\hat{\beta}_2\right)-\frac{\sigma_\mu^2}{n}-\bar{X}_2^2\,\text{var}\left(\hat{\beta}_2\right)+\bar{X}_1^2\,\text{var}\left(\hat{\beta}_1\right)+\frac{\sigma_\mu^2}{n}$$

$$-02\bar{X}_2^2\,\text{var}\left(\hat{\beta}_2\right)-02\bar{X}_2\bar{X}_1\,\text{cov}\left(\hat{\beta}_2,\hat{\beta}_1\right)$$

permission $02\bar{X}_2\,\text{cov}\left(\beta_0,\beta_2\right)=-02\bar{X}_2^2\,\text{var}\left(\hat{\beta}_2\right)-02\bar{X}_2\bar{X}_1\,\text{cov}\left(\hat{\beta}_2,\hat{\beta}_1\right)$

$$\bar{X}_2\,\text{cov}\left(\beta_0,\beta_2\right)=-\bar{X}_2^2\,\text{var}\left(\hat{\beta}_2\right)-\bar{X}_2\bar{X}_1\,\text{cov}\left(\hat{\beta}_2,\hat{\beta}_1\right)\Rightarrow$$

$$\text{cov}\left(\beta_0,\beta_2\right)=\frac{-\bar{X}_2^2\,\text{var}\left(\hat{\beta}_2\right)-\bar{X}_2\bar{X}_1\,\text{cov}\left(\hat{\beta}_2,\hat{\beta}_1\right)}{\bar{X}_2}$$

The previous two results can also be put in matrix form as follows

$$\begin{bmatrix}\text{cov}\left(\beta_0,\beta_1\right)\\ \text{cov}\left(\beta_0,\beta_2\right)\end{bmatrix}=-\begin{bmatrix}\text{var}\left(\hat{\beta}_1\right) & \text{cov}\left(\hat{\beta}_1,\hat{\beta}_2\right)\\ \text{cov}\left(\hat{\beta}_2,\hat{\beta}_1\right) & \text{var}\left(\hat{\beta}_2\right)\end{bmatrix}\begin{bmatrix}\bar{X}_1\\ \bar{X}_2\end{bmatrix}$$

If we haveK independent variables, the relationship takes the following form

$$\text{cov}\left(\beta_0,\beta_j\right)=-\sigma_\mu^2\left(X^TX\right)^{-1}\bar{X}$$

As for the value of the variance of **:Note 02** $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$ and the covariance between them $\text{cov}(\hat{\beta}_1,\hat{\beta}_2)$ using the deviation method, it is given by :the following relationship $\text{var}(B)-\text{cov}(B)=\sigma_\mu^2\times(X^TX)^{-1}$

We have the above

$$(X^TX)=\begin{bmatrix}\sum x_{i1}^2 & \sum x_{i1}x_{i2}\\ \sum x_{i1}x_{i2} & \sum x_{i2}^2\end{bmatrix}\wedge(X^TY)=\begin{bmatrix}\sum x_{i1}y_i\\ \sum x_{i2}y_i\end{bmatrix}\wedge(B)=\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}$$

We calculate the general determinant of the matrix

$$|D|=\left(\sum x_{i1}^2\right)\left(\sum x_{i2}^2\right)-\left(\sum x_{i1}x_{i2}\right)^2$$

:We extract the inverse of the matrix according to the following relationship

$$(X^TX)^{-1}=\frac{1}{|(X^TX)|}\text{adj}(X^TX)^t$$

We first extract the matrix of conjugates (minimums) as follows

$$(X^TX)^C=\begin{bmatrix}\sum x_{i2}^2 & -\sum x_{i1}x_{i2}\\ -\sum x_{i1}x_{i2} & \sum x_{i1}^2\end{bmatrix}$$

A matrix converter $(X^TX)^T$ is a matrix converter $(X^TX)^C$

$$(X^TX)^{-1}=\frac{01}{|(X^TX)|}\text{adj}(X^TX)^T\Rightarrow(X^TX)^{-1}=\frac{01}{\left(\sum x_{i1}^2\right)\left(\sum x_{i2}^2\right)-\left(\sum x_{i1}x_{i2}\right)^2}\times\begin{bmatrix}\sum x_{i2}^2 & -\sum x_{i1}x_{i2}\\ -\sum x_{i1}x_{i2} & \sum x_{i1}^2\end{bmatrix}$$

And accordingly

$$\text{var}(B)-\text{cov}(B)=\sigma_\mu^2\times(X^TX)^{-1}\Rightarrow$$

$$\text{var}(B)-\text{cov}(B)=\sigma_\mu^2\times\frac{01}{\left(\sum x_{i1}^2\right)\left(\sum x_{i2}^2\right)-\left(\sum x_{i1}x_{i2}\right)^2}\times\begin{bmatrix}\sum x_{i2}^2 & -\sum x_{i1}x_{i2}\\ -\sum x_{i1}x_{i2} & \sum x_{i1}^2\end{bmatrix}$$

So, showing the variance of $\text{var}(\hat{\beta}_1)$ and and $\text{var}(\hat{\beta}_2)$ the covariance between them $\text{cov}(\hat{\beta}_1,\hat{\beta}_2)$ is

$$var(\beta_1) = \sigma_\mu^2 \times \frac{\sum x_{i2}^2}{(\sum x_{i1}^2)(\sum x_{i2}^2) - (\sum x_{i1}x_{i2})^2}$$

$$var(\beta_2) = \sigma_\mu^2 \times \frac{\sum x_{i1}^2}{(\sum x_{i1}^2)(\sum x_{i2}^2) - (\sum x_{i1}x_{i2})^2}$$

$$cov(\beta_1, \beta_2) = -\sigma_\mu^2 \times \frac{\sum x_{i1}x_{i2}}{(\sum x_{i1}^2)(\sum x_{i2}^2) - (\sum x_{i1}x_{i2})^2}$$

Fifth: Calculating the random error variance (σ_μ^2) The theoretical formula :
for the random error variance can be derived as follows

$$\sigma_\mu^2 = \frac{\sum e_i^2}{n-k}$$

We know that $\sum e_i^2 = e_i \times e_i^T$

$$Y_i = Yi + e_i \Rightarrow e_i = Y_i - Yi$$

permission $\sum e_i^2 = (Y_i - Yi)(Y_i - Yi)^T$

We also know that $Yi = XB$
By substituting, we find

$$\sum e_i^2 = e_i \times e_i^t = (Y_i - XB)(Y_i - XB)^T = (Y_i - XB)(Y_i^T - X^T B^T)$$

$$= Y_i Y_i^T - Y_i X^T B^T - XBY_i^T + XBX^T B^T$$

and the third term represent a single value and each Since the second term ...represents the commutator of the other, then

$$Y_i X^T B^T = (Y_i X^T B^T)^T = XBY_i^T$$

permission $\sum e_i^2 = Y_i Y_i^T - 02Y_i X^T B^T + XBX^T B^T$

$$B = (X^T X)^{-1}(X^T Y)$$

We know from the above that

$$B^T = (B^T)^T = B$$

:relationship, we find Substituting it into the previous

$$\sum e_i^2 = Y_i Y_i^T - 02Y_i X^T B^T + (XX^T)(X^T X)^{-1}(X^T Y_i)B^T$$

$$= \sum Y_i^2 - 02Y_i X^T B^T + X^T Y_i B^T$$

$$= \sum Y_i^2 - Y_i X^T B^T$$

permission $\sum e_i^2 = \sum Y_i^2 - Y_i X^T B^T$

As we know that $X^TY_i = \begin{bmatrix} \sum y_i x_{i1} \\ \sum y_i x_{i2} \end{bmatrix} \times \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$

The sum of the remainders becomes equal to

$$\sum e_i^2 = \sum Y_i^2 - \beta_1 \sum y_i x_{i1} - \beta_2 \sum y_i x_{i2}$$

The random error statement becomes equal to

$$\sigma_\mu^2 = \frac{\sum e_i^2}{n-k} = \frac{\sum Y_i^2 - \beta_1 \sum y_i x_{i1} - \beta_2 \sum y_i x_{i2}}{n-k}$$

From the same data as the previous example **:Example**

Find the variance and covariance of the parameters estimated using the **-01** ?original value method

of the parameters estimated using the Find the variance and covariance **-02** ?summed value method

:the solution

Find the variance and covariance of the parameters estimated using the **-01** original value method

:We have the mathematical formula for variance and covariance as follows

$$\text{var}-\text{cov}(B) = \sigma_\mu^2 \times \left(X^TX\right)^{-1}$$

:We first calculate the variance of the random error as follows

$$\sigma_\mu^2 = \frac{\sum e_i^2}{n-k} = \frac{\sum Y_i^2 - \beta_1 \sum y_i x_{i1} - \beta_2 \sum y_1 x_{i2}}{n-k} \Rightarrow$$

$$\sigma_\mu^2 = \frac{\sum e_i^2}{n-k} = \frac{256,40 - (0,97766)(240) - (0,123730)(161,20)}{10-03} = 0,2595$$

Above, we have the inverse of the matrix with the original values

$$\left(X^TX\right)^{-1} = \frac{1}{41364}\begin{bmatrix} 48501 & 3528 & -7947 \\ 3528 & 1176 & -1500 \\ -7947 & -1500 & 2265 \end{bmatrix}$$

permission

$$\text{var}-\text{cov}(B) = \sigma_\mu^2 \times \left(X^TX\right)^{-1} \Rightarrow \text{var}-\text{cov}(B) = 0,2595 \times \frac{1}{41364}\begin{bmatrix} 48501 & 3528 & -7947 \\ 3528 & 1176 & -1500 \\ -7947 & -1500 & 2265 \end{bmatrix}$$

$$\text{var}(\beta_0) = 0,30427 \ , \ \text{cov}(\beta_0, \beta_1) = 0,02213$$

$$\text{var}(\beta_1) = 0,00737 \ , \ \text{cov}(\beta_1, \beta_2) = -0,00941$$

$$\text{var}(\beta_2) = 0,01420 \ , \ \text{cov}(\beta_2, \beta_0) = -0,04985$$

$$\beta_0 = 01,09965$$

Finding the variance and covariance of the parameters estimated using the **-02** ,summed value method: In this case, we extract the constant term $\mathrm{var}(\beta_0)$ the first term $\mathrm{var}(\beta_1)$,the second term , $\mathrm{var}(\beta_2)$:and their covariance as follows

We know from the above that

$$\mathrm{var}(\beta_1) = \sigma_\mu^2 \times \frac{\sum x_{i2}^2}{\left(\sum x_{i1}^2\right)\left(\sum x_{i2}^2\right) - \left(\sum x_{i1}x_{i2}\right)^2} \Rightarrow$$

$$\mathrm{var}(\beta_1) = 0,2595 \times \frac{117,60}{(117,60)(226,50) - (150)^2} = 0,007377$$

$$\mathrm{var}(\beta_2) = \sigma_\mu^2 \times \frac{\sum x_{i1}^2}{\left(\sum x_{i1}^2\right)\left(\sum x_{i2}^2\right) - \left(\sum x_{i1}x_{i2}\right)^2} \Rightarrow$$

$$\mathrm{var}(\beta_2) = 0,2595 \times \frac{226,50}{(117,60)(226,50) - (150)^2} = 0,01420$$

$$\mathrm{cov}(\beta_1,\beta_2) = -\sigma_\mu^2 \times \frac{\sum x_{i1}x_{i2}}{\left(\sum x_{i1}^2\right)\left(\sum x_{i2}^2\right) - \left(\sum x_{i1}x_{i2}\right)^2} \Rightarrow$$

$$\mathrm{cov}\left(\widehat{\beta_1,\beta_2}\right) = -0,2595 \times \frac{150}{(117,60)(226,50) - (150)^2} = -0,00941$$

We now extract the value of the constant term $\mathrm{var}(\beta_0)$ we know from the above :

$$\mathrm{var}(\beta_0) = \overline{X}_1^2 \,\mathrm{var}(\hat{\beta}_1) + 02\overline{X}_1\overline{X}_2 \,\mathrm{cov}(\hat{\beta}_1,\hat{\beta}_2) + \overline{X}_2^2 \,\mathrm{var}(\hat{\beta}_2) + \frac{\sigma_\mu^2}{n} \Rightarrow$$

$$\mathrm{var}(\beta_0) = (09,50)^2(0,007377) + 02(09,50)(09,80)(-0,00941) + (09,80)^2(0,01420) + \frac{0,2595}{10}$$

$$\mathrm{var}(\beta_0) = 0,30335$$

:is calculated according to the following two mechanisms The common variance

The covariance between the constant term (β_0) and the first marginal slope $(\hat{\beta}_1)$ is given by the following relationship **:**

$$\overline{X}_1 \mathrm{cov}(\beta_0,\beta_1) = -\overline{X}_1^2 \,\mathrm{var}(\hat{\beta}_1) - \overline{X}_1\overline{X}_2 \,\mathrm{cov}(\hat{\beta}_1,\hat{\beta}_2) \Rightarrow$$

$$\mathrm{cov}(\beta_0,\beta_1) = \frac{-\overline{X}_1^2 \,\mathrm{var}(\hat{\beta}_1) - \overline{X}_1\overline{X}_2 \,\mathrm{cov}(\hat{\beta}_1,\hat{\beta}_2)}{\overline{X}_1} \Rightarrow$$

$$\mathrm{cov}(\beta_0,\beta_1) = \frac{-(09,50)^2(0,007377) - (09,50)(09,80)(-0,00941)}{09,50} = 0,02213$$

The covariance between the constant term (β_0) **and the first marginal slope** $(\hat{\beta}_2)$ given by the following relationship **is**

$$\overline{X}_2 \operatorname{cov}(\beta_0,\beta_2) = -\overline{X}_2^2 \operatorname{var}(\hat{\beta}_2) - \overline{X}_2\overline{X}_1 \operatorname{cov}(\hat{\beta}_2,\hat{\beta}_1) \Rightarrow$$

$$\operatorname{cov}(\beta_0,\beta_2) = \frac{-\overline{X}_2^2 \operatorname{var}(\hat{\beta}_2) - \overline{X}_2\overline{X}_1 \operatorname{cov}(\hat{\beta}_2,\hat{\beta}_1)}{\overline{X}_2} \Rightarrow$$

$$\operatorname{cov}(\beta_0,\beta_2) = \frac{-(09,80)^2(0,01420)-(09,80)(09,50)(-0,00941)}{09,80} = -0,04977$$

We note that the results in both cases were approximately equal

Sixth: Statistical extrapolation of the estimators of the ordinary least squares method

,Knowing the distribution $(\hat{\beta}_0,\hat{\beta}_1,\hat{\beta}_2)$ it is possible to create confidence intervals and test the hypotheses made about the collapse parameters $(\beta_0,\beta_1,\beta_2)$

Estimation with a confidence interval for the parameters 01-03 $(\beta_0,\beta_1,\beta_2)$ **:**

,To form the confidence interval for the two parameters $(\hat{\beta}_0,\hat{\beta}_1,\hat{\beta}_2)$ we :distinguish two cases

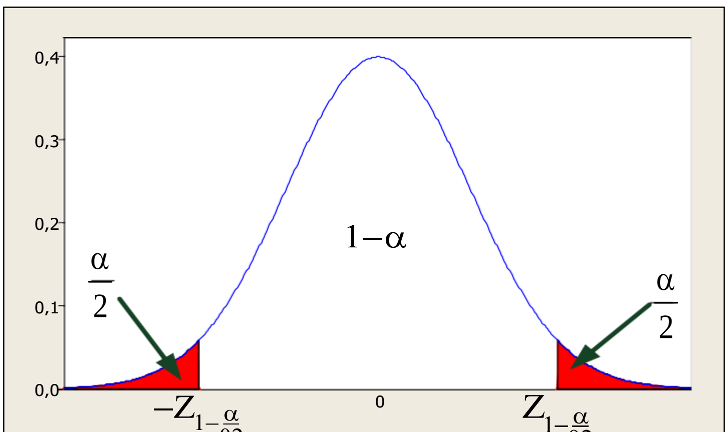
In the case of -01 $n \geq 30$ **whether** σ^2 In this case, the : **it is known or unknown**) confidence interval will be subject to the standard normal distribution Z for (.the three parameters $(\hat{\beta}_0,\hat{\beta}_1,\hat{\beta}_2)$:as follows

$$Z = \frac{\hat{\beta}_0 - E(\hat{\beta}_0)}{\sigma_{\beta_0}} = \frac{\hat{\beta}_0 - \beta_0}{\sigma_{\beta_0}} \sim N(0,1)$$

$$Z = \frac{\beta_1 - E(\beta_1)}{\sigma_{\beta_1}} = \frac{\beta_1 - \beta_1}{\sigma_{\beta_1}} \sim N(0,1)$$

$$Z = \frac{\beta_2 - E(\beta_2)}{\sigma_{\beta_2}} = \frac{\beta_2 - \beta_2}{\sigma_{\beta_2}} \sim N(0,1)$$

α) The confidence interval is calculated based on the level of significance (:according to the following figure



It is clear from the figure that the confidence interval is

$$P\left(-Z_{1-\frac{\alpha}{2}} \le Z \le Z_{1-\frac{\alpha}{2}}\right) = 1-\alpha$$

Confidence range for the parameter 01-01 β_0 By replacing the value of **:**Z in range equation, we find the confidence

$$P\left(\beta_0 - Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta_0} \le \beta_0 \le \beta_0 + Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta_0}\right) = 1-\alpha$$

:The last equation is written briefly as follows

$$IC(\beta_0)_{1-\alpha} = \beta_0 \pm T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_0}$$

Confidence range for the parameter 02-01 β_1 By replacing the value of **:**Z in equation, we find the confidence range

$$P\left(\beta_1 - Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta_1} \le \beta_1 \le \beta_1 + Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta_1}\right) = 1-\alpha$$

:The last equation is written briefly as follows

$$IC(\beta_1)_{1-\alpha} = \hat{\beta}_1 \pm Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta_1}$$

Confidence range for the parameter 03-01 β_2 By replacing the value of **:**Z in find the confidence range equation, we

$$P\left(\beta_2 - Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2} \le \beta_2 \le \beta_2 + Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2}\right) = 1-\alpha$$

:The last equation is written briefly as follows

$$IC(\beta_2)_{1-\alpha} = \hat{\beta}_2 \pm Z_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2}$$

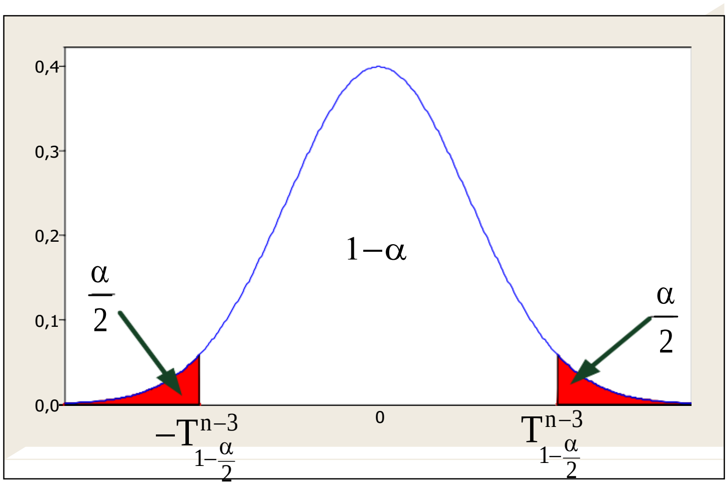
In the case -02 $n < 30$ **of** σ^2 In this case, the confidence interval will : **unknown**) be subject to the StudentT .distribution for the three features ($\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$ as :follows

$$T=\frac{\hat{\beta}_0-E(\hat{\beta}_0)}{\sigma_{\hat{\beta}_0}}=\frac{\hat{\beta}_0-\beta_0}{\sigma_{\hat{\beta}_0}}\sim t\left(0,\frac{n}{n-2}\right)$$

$$T=\frac{\hat{\beta}_1-E(\hat{\beta}_1)}{\sigma_{\hat{\beta}_1}}=\frac{\hat{\beta}_1-\beta_1}{\sigma_{\hat{\beta}_1}}\sim t\left(0,\frac{n}{n-2}\right)$$

$$T=\frac{\hat{\beta}_2-E(\hat{\beta}_2)}{\sigma_{\hat{\beta}_2}}=\frac{\hat{\beta}_2-\beta_2}{\sigma_{\hat{\beta}_2}}\sim t\left(0,\frac{n}{n-2}\right)$$

α) calculated based on the level of significance (The confidence interval is :according to the following figure



It is clear from the figure that the confidence interval is

$$P\left(-T^{n-3}_{1-\frac{\alpha}{2}}\le T\le T^{n-3}_{1-\frac{\alpha}{2}}\right)=1-\alpha$$

Confidence range for the parameter 01-02 β_0 By replacing the value of **:**T in the confidence range equation, we find

$$P\left(\beta_0-T^{n-3}_{1-\frac{\alpha}{2}}\times\sigma_{\beta_0}\le\beta_0\le\beta_0+T^{n-3}_{1-\frac{\alpha}{2}}\times\sigma_{\beta_0}\right)=1-\alpha$$

:The last equation is written briefly as follows

$$IC(\beta_0)_{1-\alpha}=\hat{\beta}_0\pm T^{n-3}_{1-\frac{\alpha}{2}}\times\sigma_{\beta_0}$$

Confidence range for the parameter 02-02 β_1 By replacing the value of **:**T in the confidence range equation, we find

$$P\left(\beta_1-T^{n-3}_{1-\frac{\alpha}{2}}\times\sigma_{\beta_1}\le\beta_1\le\beta_1+T^{n-3}_{1-\frac{\alpha}{2}}\times\sigma_{\beta_1}\right)=1-\alpha$$

:The last equation is written briefly as follows

$$IC(\beta_1)_{1-\alpha}=\hat{\beta}_1\pm T^{n-3}_{1-\frac{\alpha}{2}}\times\sigma_{\beta_1}$$

Confidence range for the parameter 03-01 β_2 By replacing the value of **:**T in the confidence range equation, we find

$$P\left(\beta_2 - T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2} \leq \beta_2 \leq \beta_2 + T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2}\right) = 1-\alpha$$

:The last equation is written briefly as follows

$$IC(\beta_2)_{1-\alpha} = \beta_2 \pm T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2}$$

the same data as the previous example From **:Example**

:Required

Find the confidence interval for the three parameters at a significance level **-01** of 01% ?

Since the sample size studied is less than 30 and the population **:Solution** distribution variance is unknown, we use the Student

The confidence interval for the parameter **-01** β_0 is at a significance level of 01 %.

:We know that the confidence interval is given by the following relationship

$$P\left(\beta_0 - T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_0} \leq \beta_0 \leq \beta_0 + T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_0}\right) = 1-\alpha$$

As we know from the above $var(\beta_0) = 0,30427$ that $\beta_0 = 01,09965$

We extract the value of Student at a degree of freedom of 07 and a significance level of 01% as follows $T^{n-3}_{1-\frac{\alpha}{2}} \Rightarrow T^{07}_{0.995} = 03,499$

$$P\left(\beta_0 - T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_0} \leq \beta_0 \leq \beta_0 + T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_0}\right) = 1-\alpha \Rightarrow$$

$$P\left(01,09965 - (03,499) \times \sqrt{0,30427} \leq \beta_0 \leq 01,09965 + (03,499) \times \sqrt{0,30427}\right) = 0,99$$

$$P\left(-0,8304 \leq \beta_0 \leq 03,0297\right) = 0,99$$

The confidence interval for the parameter **-2** β_1 is at a significance level of 01%.

:We know that the confidence interval is given by the following relationship

$$P\left(\beta_1 - T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_1} \leq \beta_1 \leq \beta_1 + T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_1}\right) = 1-\alpha$$

As we know from the above $var(\beta_1) = 0,00737$ that $\beta_1 = 0,97766$

We extract the value of Student at a degree of freedom of 07 and a significance level of 01% as follows $T^{n-3}_{1-\frac{\alpha}{2}} \Rightarrow T^{07}_{0.995} = 03,499$

$$P\left(\beta_1 - T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_1} \le \beta_1 \le \beta_1 + T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_1}\right) = 1-\alpha \Rightarrow$$

$$P\left(0,97766 - (03,499) \times \sqrt{0,00737} \le \beta_1 \le 0,97766 + (03,499) \times \sqrt{0,00737}\right) = 0,99$$

$$P\left(0,6773 \le \beta_1 \le 01,2780\right) = 0,99$$

The confidence interval for the parameter **-3** β_2 is at a significance level of 01 %.

:We know that the confidence interval is given by the following relationship

$$P\left(\beta_2 - T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2} \le \beta_2 \le \beta_2 + T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2}\right) = 1-\alpha$$

As we know from the above $var\left(\beta_2\right) = 0,01420$ that $\beta_2 = 0,12373$

We extract the value of Student at a degree of freedom of 07 and a significance level of 01% as follows $T^{n-3}_{1-\frac{\alpha}{2}} \Rightarrow T^{07}_{0.995} = 03,499$

$$P\left(\beta_2 - T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2} \le \beta_2 \le \beta_2 + T^{n-3}_{1-\frac{\alpha}{2}} \times \sigma_{\beta_2}\right) = 1-\alpha \Rightarrow$$

$$P\left(0,12373 - (03,499) \times \sqrt{0,01420} \le \beta_1 \le 0,12373 + (03,499) \times \sqrt{0,01420}\right) = 0,99$$

$$P\left(-0,2932 \le \beta_2 \le 0,5407\right) = 0,99$$

using statistical and measurement Seventh: Study the validity of the model tools

To study the validity of the model, we follow the following steps

Coefficient of determination 01-07 $\left(R^2\right)$ As we saw previously in simple : linear regression, the sum of the squares of the total deviations $\sum\left(Y_i - \overline{Y}\right)^2$ is divided into two parts. The first represents the sum of the squares of the (explanatory deviations (explained $\sum\left(\hat{Y}_i - \overline{Y}\right)^2$ and the second represents the , explanatory deviations (not explained) (the sum of -sum of the squares of the non .(the residuals $\sum\left(Y_i - \hat{Y}_i\right)^2$:That is

$$\frac{\sum\left(Y_i - \overline{Y}\right)^2}{n} = \frac{\sum\left(\hat{Y}_i - \overline{Y}\right)^2}{n} + \frac{\sum\left(Y_i - \hat{Y}_i\right)^2}{n}$$

$$SST = SSR + SSE$$

) Total varianceSST) explained variance = (SSR unexplained variance + ( ) or residual varianceSSE (

The total variance formula can also be written in terms of explanatory variables as follows

$$\sum\left(Y_i-\overline{Y}\right)^2=\sum\left(\hat{Y}_i-\overline{Y}\right)^2+\sum\left(Y_i-\hat{Y}_i\right)^2\Rightarrow$$

$$\sum\left(Y_i-\overline{Y}\right)^2=Y_iX^TB^T+\sum e_i^2$$

$$\sum\left(Y_i-\overline{Y}\right)^2=\hat{\beta}_1\sum y_ix_{i1}+\hat{\beta}_2\sum y_ix_{i2}+\sum e_i^2$$

The coefficient of determination is given by the following mathematical relationship

$$R^2=\frac{SSR}{SST}=\frac{\sum\left(\hat{Y}_i-\overline{Y}\right)^2}{\sum\left(Y_i-\overline{Y}\right)^2}=\frac{Y_iX^TB^T}{\sum\left(Y_i-\overline{Y}\right)^2}=\frac{\hat{\beta}_1\sum y_ix_{i1}+\hat{\beta}_2\sum y_1x_{i2}}{\sum\left(Y_i-\overline{Y}\right)^2}$$

It can also be given by the following relationship

$$R^2=\frac{SSR}{SST}=\frac{SST-SSE}{SST}=01-\frac{SSE}{SST}\Rightarrow R^2=01-\frac{\sum\left(Y_i-\hat{Y}\right)^2}{\sum\left(Y_i-\overline{Y}\right)^2}$$

$$R^2=01-\frac{\sum e_i^2}{\sum Y_i^2}=01-\frac{\sum Y_i^2-\beta_1\sum y_ix_{i1}-\beta_2\sum y_ix_{i2}}{\sum Y_i^2}$$

As a final relationship, it can be deduced from the previous relationship as follows

$$R^2=\frac{\beta_1\sum y_ix_{i1}+\beta_2\sum y_ix_{i2}}{\sum Y_i^2}$$

Where x_{i1} and x_{i2} and y_i are the shortened values

The coefficient of determination is limited to between zero and one $0\le R^2\le 01$

The coefficient of determination can also be expressed $\left(R^2\right)$ in terms of simple :correlation coefficients as follows

$$R^2=\frac{r^2_{x_1y_i}+r^2_{x_2y_i}-02r_{x_1y_i}\times r_{x_2y_i}\times r_{x_1x_2}}{01-r^2_{x_1x_2}}$$

where

$r^2_{x_1y}$ The simple correlation coefficient between : x_1 and y

$r^2_{x_2y}$ The simple correlation coefficient between : x_2 and y

$r^2_{x_1x_2}$ The simple correlation coefficient between : x_1 and x_2

statistical relationship for simple correlation coefficients is given as follows The

$$r_{x_1y_i} = \frac{\sum x_1y_i}{\sqrt{\sum x_1^2} \times \sqrt{\sum y_i^2}}$$

$$r_{x_2y_i} = \frac{\sum x_2y_i}{\sqrt{\sum x_2^2} \times \sqrt{\sum y_i^2}}$$

$$r_{x_1x_2} = \frac{\sum x_1x_2}{\sqrt{\sum x_1^2} \times \sqrt{\sum x_2^2}}$$

Modified coefficient of determination 02-07 $\left(\overline{R}^2\right)$ Adding independent : variables to the equation leads to raising the value $\left(R^2\right)$ because the value of the denominator is constant and the value of the numerator changes. However, continuing to add independent variables will lead to a decrease in the degrees of freedom, corrected coefficient of which requires extracting the modified or :determination as follows

$$\overline{R}^2 = 01 - \left[\left(01 - R^2\right) \times \frac{n-01}{n-k}\right]$$

The partial correlation coefficient **:Partial correlation coefficients 03-07** measures the strength of the linear relationship between two variables after isolating or excluding the effect of other variables. The difference between the ion coefficient is that simple linear correlation coefficient and the partial correlat the first measures the strength and direction of the relationship between two variables within the effects of other variables. While the second measures the uding strength and direction of the relationship between two variables after excl ,the influence of other variables. For example, if we have three variables (X_1, X_2, Y) it is possible to measure the correlation between any two of them t while isolating the effect of the third by using the partial correlation coefficien ,between the variable Y for X_1 example, after excluding the effect of the .variable X_2 The partial correlation coefficient is symbolized by the symbol the next

ρ_{YX_1,X_2} Partial correlation coefficients are divided into two types

order -The first : **order correlation coefficient-Partial first 01-07-03** correlation coefficient is calculated between Y and X_1 After excluding the effect of the variable X_2 which is represented by the symbol , r_{YX_1,X_2} According to the :following formula

$$r_{YX_1,X_2} = \frac{r_{YX_1} - r_{X_1X_2} \times r_{YX_2}}{\sqrt{01 - r_{X_1X_2}^2} \times \sqrt{01 - r_{YX_2}^2}}$$

Where r_{YX_1} , $r_{X_1X_2}$, r_{YX_2} are the simple correlation coefficients between pairs of variables (Y, X_1) and (Y, X_2) and (X_1, X_2) respectively. Likewise, the partial correlation coefficient can be calculated between Y and X_2 After excluding the ,effect of the variable X_1 :it is as follows

$$r_{YX_2,X_1}=\frac{r_{YX_2}-r_{X_1X_2}\times r_{YX_1}}{\sqrt{01-r^2_{X_1X_2}}\times\sqrt{01-r^2_{YX_1}}}$$

The effect of the two variables can also be measured (X_1,X_2) after recovering the dependent variable Y as follows

$$r_{X_1X_2,Y}=\frac{r_{X_1X_2}-r_{YX_2}\times r_{YX_1}}{\sqrt{01-r^2_{YX_1}}\times\sqrt{01-r^2_{YX_2}}}$$

order -The second **:order partial correlation coefficient-Second 02-03-07** order partial -partial correlation coefficient is considered an extension of the first correlation coefficient, as the partial correlation coefficient takes between Y and X_1 After excluding the effect of the variable X_2 , X_3 :in the following formula

$$r_{YX_1,X_2X_3}=\frac{r_{YX_1,X_2}-r_{YX_3,X_2}\times r_{X_1X_3,X_2}}{\sqrt{01-r^2_{YX_3,X_2}}\times\sqrt{01-r^2_{X_1X_3,X_2}}}=\frac{r_{YX_1,X_3}-r_{YX_2,X_3}\times r_{X_1X_2,X_3}}{\sqrt{01-r^2_{YX_2,X_3}}\times\sqrt{01-r^2_{X_1X_2,X_3}}}$$

If the partial correlation coefficient between- Y and X_2 After excluding the effect of the variable X_1 , X_3 :in the following formula

$$r_{YX_2,X_1X_3}=\frac{r_{YX_2,X_1}-r_{YX_3,X_1}\times r_{X_2X_3,X_1}}{\sqrt{01-r^2_{YX_3,X_1}}\times\sqrt{01-r^2_{X_2X_3,X_1}}}=\frac{r_{YX_2,X_3}-r_{YX_1,X_3}\times r_{X_1X_2,X_3}}{\sqrt{01-r^2_{YX_1,X_3}}\times\sqrt{01-r^2_{X_1X_2,X_3}}}$$

If the partial correlation coefficient between- Y and X_3 After excluding the effect of the variable X_1 , X_2 :in the following formula

$$r_{YX_3,X_1X_2}=\frac{r_{YX_3,X_1}-r_{YX_2,X_1}\times r_{X_2X_3,X_1}}{\sqrt{01-r^2_{YX_2,X_1}}\times\sqrt{01-r^2_{X_2X_3,X_1}}}=\frac{r_{YX_3,X_2}-r_{YX_1,X_2}\times r_{X_3X_1,X_2}}{\sqrt{01-r^2_{YX_1,X_2}}\times\sqrt{01-r^2_{X_3X_1,X_2}}}$$

From the same data as the previous example : **Example**

:Required

determination Calculate the coefficient of **-01** R^2 And the modified definition $\overline{R}^2$?

Calculate the simple correlation coefficients, then recalculate the coefficient **-02** ?of determination, and what do you conclude

Calculate the partial correlation coefficients between **-03** Y and X_1 After excluding the effect of the variable X_2 and between Y and X_2 After excluding the effect of the variable X_1 ?what do you conclude ,

:the solution

Calculate the coefficient of determination - **01** R^2 And the modified definition $\overline{R}^2$

Calculate the coefficient of determination R^2 using the following relationships **:**

$$R^2=\frac{\beta_1\sum y_i x_{i1}+\beta_2\sum y_i x_{i2}}{\sum Y_i^2}$$

We have the above

$\sum y_i^2=256,40$,$\sum x_{i1}y_i=240$,$\sum x_{i2}y_i=161,20$,$\sum x_{i1}x_{i2}=150$,$\sum x_{i1}^2=226,50$,$\sum x_{i2}^2=117,60$

$\beta_1=0.97766$,$\beta_2=0,12373$

permission

$$R^2=\frac{\beta_1\sum y_i x_{i1}+\beta_2\sum y_i x_{i2}}{\sum Y_i^2}\Rightarrow R^2=\frac{(0,97766)(240)+(0,12373)(161,20)}{(256,40)}=0,9929$$

That is **:Interpretation**, 2.9 99% of the changes in the dependent variable Y_i could be explained by changes in the independent variable X_i and the remaining 0.71% .by other factors

Calculate the modified coefficient of determination - $\overline{R}^2$ by applying the : following law

$$\overline{R}^2=01-\left[\left(01-R^2\right)\times\frac{n-01}{n-k}\right]\Rightarrow\overline{R}^2=01-\left[\left(01-0,9929\right)\times\frac{10-01}{10-03}\right]=0,9908$$

Calculating simple correlation coefficients and then calculating the **-02** coefficient of determination: We know from the above that terms of simple correlation The coefficient of determination is given in coefficients according to the following relationship

$$R^2=\frac{r_{x_1y_i}^2+r_{x_2y_i}^2-02r_{x_1y_i}\times r_{x_2y_i}\times r_{x_1x_2}}{01-r_{x_1x_2}^2}$$

:We extract the simple coefficients as follows

$$r_{x_1y_i}=\frac{\sum x_1y_i}{\sqrt{\sum x_1^2}\times\sqrt{\sum y_i^2}}\Rightarrow r_{x_1y_i}=\frac{240}{\sqrt{226,50}\times\sqrt{256,40}}=0,9959$$

$$r_{x_2y_i}=\frac{\sum x_2y_i}{\sqrt{\sum x_2^2}\times\sqrt{\sum y_i^2}}\Rightarrow r_{x_2y_i}=\frac{161,20}{\sqrt{117,60}\times\sqrt{256,40}}=0,9283$$

$$r_{x_1x_2}=\frac{\sum x_1x_2}{\sqrt{\sum x_1^2}\times\sqrt{\sum x_2^2}}\Rightarrow r_{x_1x_2}==\frac{150}{\sqrt{226,50}\times\sqrt{117,60}}=0,9190$$

$$R^2=\frac{r_{x_1y_i}^2+r_{x_2y_i}^2-02r_{x_1y_i}\times r_{x_2y_i}\times r_{x_1x_2}}{01-r_{x_1x_2}^2}\Rightarrow$$

permission

$$R^2=\frac{(0,9959)^2+(0,9283)^2-02(0,9959)(0,9283)(0,9190)}{01-(0,9190)^2}=0,9929$$

.It is the same result as before

Chapter three : Multiple Linear Regression

Calculate the partial correlation coefficients between **-03** Y and X_1 After excluding the effect of the variable X_2 and between Y and X_2 excluding After the effect of the variable X_1

Calculate the coefficients of partial correlations between **01-03** Y and X_1 After excluding the effect of the variable X_2

$$r_{YX_1,X_2} = \frac{r_{YX_1} - r_{X_1X_2} \times r_{YX_2}}{\sqrt{01 - r^2_{X_1X_2}} \times \sqrt{01 - r^2_{YX_2}}} \Rightarrow r_{YX_1,X_2} = \frac{0,9959 - 0,9283 \times 0,9190}{\sqrt{01 - (0,9283)^2} \times \sqrt{01 - (0,9190)^2}} = 0,9740$$

Calculate the coefficients of partial correlations between **02-03** Y and X_2 After excluding the effect of the variable X_1

$$r_{YX_2,X_1} = \frac{r_{YX_2} - r_{X_1X_2} \times r_{YX_1}}{\sqrt{01 - r^2_{X_1X_2}} \times \sqrt{01 - r^2_{YX_1}}} \Rightarrow r_{YX_2,X_1} = \frac{0,9283 - 0,9959 \times 0,9190}{\sqrt{01 - (0,9959)^2} \times \sqrt{01 - (0,9190)^2}} = 0,3664$$

The independent variable **:Conclusion** X_1 contributes more to the explanatory power of the model than the independent variable X_2

$$r_{X_1X_2,Y} = \frac{r_{X_1X_2} - r_{YX_2} \times r_{YX_1}}{\sqrt{01 - r^2_{YX_1}} \times \sqrt{01 - r^2_{YX_2}}} \Rightarrow r_{X_1X_2,Y} = \frac{0,9190 - 0,9283 \times 0,9959}{\sqrt{01 - (0,9959)^2} \times \sqrt{01 - (0,9283)^2}} = -0,1633$$

T) The Student test : **(test**T is used to evaluate the significance of the effect (of the independent variables $(X_1, X_2, X_3, ..., X_k)$ .on the dependent variable (Y_i) and a degree of α In the multiple regression model at a significance level of) freedomnk tailed test is used, i.e. on the right and left of the Student -a two ,( ) probability distributionT .This test is based on two types of hypotheses .(

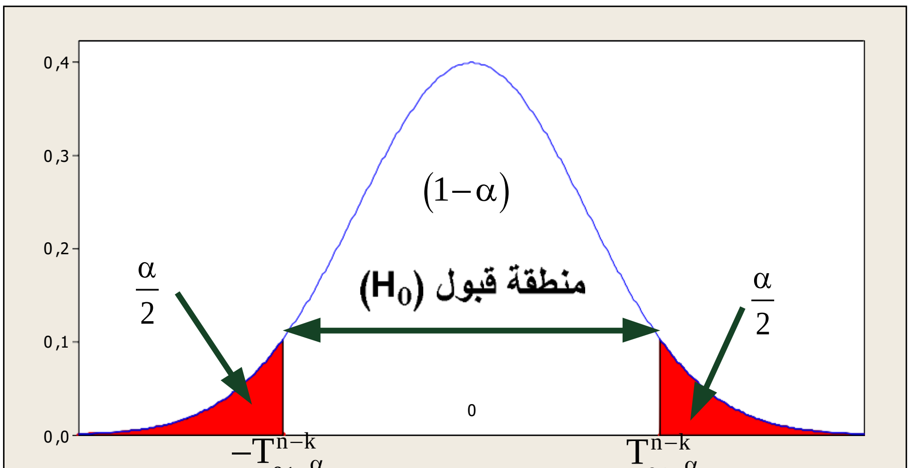
There is no relationship between the dependent variable **:Null hypothesis** (Y_i) and the independent variables (X_i) i.e

$$H_0 : \beta_1 = \beta_2 = \beta_3 = ... = \beta_k = 0$$

A relationship between the dependent variable There is **:Alternative hypothesis** (Y_i) and the independent variables (X_i) i.e

$$H_1 : \beta_1 \neq \beta_2 \neq \beta_3 \neq .... \neq \beta_k \neq 0$$

The following figure shows the rejection and acceptance method



Knowing the sample sizen the Student tabular α, and the significance level :value can be extracted according to the following formula

$$T_{tab} = T^{n-k}_{01-\frac{\alpha}{2}} = T^{n-03}_{01-\frac{\alpha}{2}}$$

:where
n sample size :
K the number of estimated features and in our case : $\left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2\right)$
Then we calculate the calculated Student formula for each of the two parameters $\hat{\beta}_1$ as $\hat{\beta}_2$ follows

For the parameter $\hat{\beta}_1$:according to the following formula $T_{cal} = \dfrac{\hat{\beta}_1 - E\left(\hat{\beta}_1\right)}{\sigma_{\beta_1}}$

For the parameter β_2 :according to the following formula $T_{cal} = \dfrac{\hat{\beta}_2 - E\left(\hat{\beta}_2\right)}{\sigma_{\hat{\beta}_2}}$

The decision will be made to reject H_0 or accept H_1 :if $T_{cal} > T_{tab}$ or $-T_{cal} < -T_{tab}$

The decision is to accept H_0 or reject H_1 :if $-T_{tab} < T_{cal} < +T_{tab}$

) (Overall study of the model (Fisher's test 05-07F This test aims to **:(** determine the significance of the linear relationship between the independent variables $\left(X_1, X_2, X_3, \ldots\ldots\ldots, X_k\right)$ on the dependent variable $\left(Y_i\right)$ This test is done .) and a degree of freedomα at a significance level of nk tailed test -and the one ,( ) .is used for the Fisher probability distributionF Through the result of this (ect on the dependent variable are test, independent variables that have no eff excluded from the model even though the estimated value of the coefficients of these variables differs from zero. In the overall study using the Fisher test, we Partial .have two types of test: the Fisher test and the Fisher test

This **:(Testing the overall significance of the model (Fisher's test 01-05-07** :test is based on two types of hypotheses

There is no relationship between all independent variables **:Null hypothesis** $\left(X_i\right)$ and the dependent $\left(Y_i\right)$ variable

$H_0 : \beta_1 = \beta_2 = \beta_3 = ... = \beta_k = 0$

There is a significant relationship between the **:Alternative hypothesis** independent variables (X_i) and the dependent variable (Y_i) i.e ,

$H_1 : \beta_1 \neq \beta_2 \neq \beta_3 \neq .... \neq \beta_k \neq 0$

Knowing the sample sizen the Fisher tabular value α, and the significance level :can be extracted according to the following formula

$F_{tab} = F_{01-\alpha, K-1, n-k} = F_{01-\alpha, V_1, V_2}$

Then we calculate Fisher's formula :As follows

$$F_{cal} = \frac{\frac{SSR}{K-1}}{\frac{SSE}{n-k}} = \frac{\frac{\sum\left(Y_i - \overline{Y}\right)^2}{K-1}}{\frac{\sum\left(Y_i - Y_i\right)^2}{n-k}} = \frac{\frac{Y_i X^T B^T}{K-1}}{\frac{\sum e_i^2}{n-k}} = \frac{\frac{Y_i X^T B^T}{K-1}}{\frac{\sum Y_i^2 - Y_i X^T B^T}{n-k}}$$

$$= \frac{(n-k)\left(\hat{\beta}_1 \sum y_i x_{i1} + \hat{\beta}_2 \sum y_1 x_{i2}\right)}{(K-1)\left(\sum Y_i^2 - \hat{\beta}_1 \sum y_i x_{i1} - \hat{\beta}_2 \sum y_1 x_{i2}\right)}$$

:So the calculated Fisher formula is $F_{cal} = \frac{(n-k)\left(\hat{\beta}_1 \sum y_i x_{i1} + \hat{\beta}_2 \sum y_1 x_{i2}\right)}{(K-1)\left(\sum Y_i^2 - \hat{\beta}_1 \sum y_i x_{i1} - \hat{\beta}_2 \sum y_1 x_{i2}\right)}$

) It is also possible to find the relationship between the Fisher distribution house F and the coefficient of determination (R^2 as follows

We know that $R^2 = \frac{SSR}{SST} = \frac{Y_i X^T B^T}{\sum Y_i^2}$

and that

$$F_{cal} = \frac{\frac{SSR}{K-1}}{\frac{SSE}{n-k}} ....(01)$$

permission $R^2 \times SST = SSR \Rightarrow R^2 \times \sum Y_i^2 = Y_i X^T B^T$.............(02)

$$SSE = \sum e_i^2 = \sum Y_i^2 - Y_i X^T B^T \Rightarrow SSE = \sum e_i^2 = \sum Y_i^2 - R^2 \times \sum Y_i^2$$

$$\Rightarrow SSE = \sum e_i^2 = \sum Y_i^2 \left(01 - R^2\right) ....(03)$$

formula, we find Substituting (02) and (03) into Fisher's

$$F_{cal} = \frac{\frac{SSR}{K-1}}{\frac{SSE}{n-k}} = \frac{\frac{R^2 \times \sum Y_i^2}{K-1}}{\frac{\sum Y_i^2 \left(01 - R^2\right)}{n-k}} = \frac{(n-k) \times R^2}{(K-1) \times \left(01 - R^2\right)}$$

$$F_{cal} = \frac{(n-k)\times R^2}{(K-1)\times(01-R^2)}$$

:where

n sample size :

K the number of estimated features and in our case : $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$

The decision will be made to reject H_0 or accept H_1 :if $F_{cal} > F_{tab}$

The decision is to accept H_0 or reject H_1 :if $F_{cal} < F_{tab}$

) Analysis of variance tableANOVA In order to determine the effect of both :(X_1 and X_2 on the dependent variable Y_i it is necessary to create an analysis of , variance table to show the effect of the independent variables X_1 and X_2 on the .model

Source of variance	The sum of the squares of the deviations	Degree of freedom	Mean square error	FisherF value
The deviations shown by X_1 and X_2	$SSR = \sum(\hat{Y}_i - \overline{Y})^2$ $= Y_i X^T B^T$ $= \hat{\beta}_1 \sum y_i x_{i1} + \hat{\beta}_2 \sum y_1 x_{i2}$	**K-01**	$MSR = \frac{SSR}{k-01}$ $= \frac{\sum(Y_i - \overline{Y})^2}{k-01}$	$F_{cal} = \frac{SSR}{SSE}$ $= \frac{\sum(Y_i - \overline{Y})^2 / K-1}{\sum(Y_i - Y_i)^2 / n-1}$ $= \frac{(n-k)(Y_i X^T B^T)}{(n-1)(\sum Y_i^2 - Y_i X^T B^T)}$
Deviations not shown	$SSE = \sum(Y_i - Y_i)^2 = \sum e_i^2$ $= \sum Y_i^2 - Y_i X^T B^T$	**n-K**	$MSE = \frac{SSE}{n-k}$ $= \frac{\sum(Y_i - Y_i)^2}{n-k}$	
Total deviations	$SST = \sum(Y_i - \overline{Y})^2 = \sum Y_i^2$	**n-01**		

individually on Selecting the effect of each independent variable 02-07-05 :(the dependent variable (Fisher's partial test

Fisher's partial test shows how to test the effect of each independent variable individually on the dependent variable. Suppose we have the following multiple :linear regression model $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \mu_i$

Continuing the partial test after confirming the joint significant effect of the variablesX_1 and$X$ is carried out in two stages $_2$

The first stage: the effect of the variableX_1 on the dependent variableY_i

testing the independent effect of the independent variable For the purpose of X_1 In the dependent variable Y_i It is necessary to know the amount of the increase achieved in the value of the sum of the squares of the deviations shown by the regression line of the dependent variable Y_i on the independent variable X_2 as a result of adding the independent variable X_1 to the function. This is done by ,assuming a model that includes the variable X_2 ,that is

$$Y_i = \beta_0 + \beta_2 X_{i2} + \mu_i$$

:hypotheses This test is done using the following two

There is no relationship between the independent variable **:Null hypothesis** (X_1) and the dependent (Y_i) variable $H_0 : \beta_1 = 0$

There is a significant relationship between the **:Alternative hypothesis** independent variable (X_1) and the dependent variable (Y_i) i.e , $H_1 : \beta_1 \neq 0$

We use an analysis of variance table to calculate the effect (X_1) individually on the dependent variable

Source of variance	The sum of the squares of the deviations	Degree of freedom	Mean square error	FisherF value
The deviation described before X_2	$SSR = \beta_2 \sum y_1 x_{i2}$	01		$F_{cal} = \frac{SSR}{SSE}$
The deviation described before X_1	$SSR = \beta_1 \sum y_i x_1$ $= Y_i X^T B^T - \beta_2 \sum y_1 x_2$	01	$MSR = \beta_1 \sum y_i x_1$ $= Y_i X^T B^T - \beta_2 \sum y_1 x_2$	
The deviations shown by X_1 and X_2	$SSR = \sum(\hat{Y}_i - \overline{Y})^2$ $= Y_i X^T B^T$ $= \beta_1 \sum y_i x_{i1} + \beta_2 \sum y_i x_{i2}$	k-01		
Deviations not shown	$SSE = \sum(Y_i - Y_i)^2 = \sum e_i^2$ $= \sum Y_i^2 - Y_i X^T B^T$	nk	$MSE = \frac{SSE}{n-k}$ $= \frac{\sum(Y_i - Y_i)^2}{n-k}$	
Total deviations	$SST = \sum(Y_i - \overline{Y})^2 = \sum Y_i^2$	n-01		

The second stage: the effect of the variable X_2 on the dependent variable Y_i

testing the independent effect of the independent variable For the purpose of X_2 In the dependent variable Y_i The amount of increase achieved in the value of the sum of the squares of the deviations shown by the regression line of the dependent variable Y_i on the independent variable X_1 as a result of adding the independent variable X_2 to the function must be known. This is done by ,assuming a model that includes the variable X_1 meaning that

Chapter three : Multiple Linear Regression

$$Y_i = \beta_0 + \beta_1 X_{i1} + \mu_i$$

:the following two hypotheses This test is done using

There is no relationship between the independent variable **:Null hypothesis** (X_2) and the dependent (Y_i) variable $H_0 : \beta_2 = 0$

There is a significant relationship between the **:Alternative hypothesis** independent variable (X_2) and the dependent variable (Y_i) i.e , $H_1 : \beta_2 \neq 0$

We use an analysis of variance table to calculate the effect (X_2) individually on the dependent variable

Source of variance	**The sum of the squares of the deviations**	**Degree of freedom**	**Mean square error**	**FisherF value**
The deviation described before X_1	$SSR = \beta_1 \sum y_i x_1$	**01**		
The deviation described before X_2	$SSR = \beta_2 \sum y_i x_2$ $= Y_i X^T B^T - \beta_1 \sum y_1 x_1$	**01**	$MSR = \beta_2 \sum y_i x_2$ $= Y_i X^T B^T - \beta_1 \sum y_1 x_1$	$F_{cal} = \frac{SSR}{SSE}$
The deviations shown by X_1 and X_2	$SSR = \sum (\hat{Y}_i - \overline{Y})^2$ $= Y_i X^T B^T$ $= \beta_1 \sum y_i x_{i1} + \beta_2 \sum y_i x_{i2}$	**k-01**		
Deviations not shown	$SSE = \sum (Y_i - Y_i)^2 = \sum e_i^2$ $= \sum Y_i^2 - Y_i X^T B^T$	**nk**	$MSE = \frac{SSE}{n-k}$ $= \frac{\sum (Y_i - Y_i)^2}{n-k}$	
Total deviations	$SST = \sum (Y_i - \overline{Y})^2 = \sum Y_i^2$	**n-01**		

From the same data as the previous example : **Example**

:Required

Study the partial validity of the model using the Student distribution at a **-01** significance level of 05% ?

Study the overall validity of the model using Fisher's distribution at a - **02** significance level of 05% With setting an analysis of variance table ?

Test the effect **-03** (X_1) individually at a 05 (Y_i) % significance level using an ?analysis of variance table

Test the effect **-04** (X_2) individually at a 05 (Y_i) % significance level using an ?analysis of variance table

:the solution

Partial study of the validity of the model at a significance level of 05 **-01**%

:We have the following two hypotheses

There is no relationship between the dependent variable **:Null hypothesis** (Y_i) and the independent variables (X_{i1}, X_{i2}) i.e

$H_0 : \beta_1 = \beta_2 = 0$

A relationship between the dependent variable There is **:Alternative hypothesis** (Y_i) and the independent variables (X_{i1}, X_{i2}) i.e

$H_1 : \beta_1 \neq \beta_2 \neq 0$

For the parameter -01 $\hat{\beta}_1$ we extract the calculated and tabulated Student : value

We know that $T_{tab} = T^{n-k}_{01-\frac{\alpha}{2}} = T^{n-03}_{01-\frac{\alpha}{2}} \Rightarrow T_{tab} = T^{07}_{0,975} = 02,365$

We know that

$$T_{cal} = \frac{\beta_1 - E(\beta_1)}{\sigma_{\beta_1}} \Rightarrow T_{cal} = \frac{0,97766 - 0}{\sqrt{0,00737}} = 11,388$$

Since $T_{cal} > T_{tab}$ we reject H_0 and accept H_1 that is, there is a relationship , between the dependent variable (Y_i) and the independent variable (X_{i1})

For the parameter -02 $\hat{\beta}_2$ we extract the calculated and tabulated Student : value

We know that $T_{tab} = T^{n-k}_{01-\frac{\alpha}{2}} = T^{n-03}_{01-\frac{\alpha}{2}} \Rightarrow T_{tab} = T^{07}_{0,975} = 02,365$

We know that

$$T_{cal} = \frac{\beta_2 - E(\beta_2)}{\sigma_{\beta_2}} \Rightarrow T_{cal} = \frac{0,12373 - 0}{\sqrt{0,01420}} = 01,0383$$

Since $-T_{tab} < T_{cal} < +T_{tab}$ we accept H_0 and reject H_1 that is, there is no , relationship between the dependent variable (Y_i) and the independent variable (X_{i2})

Study the overall validity of the model using Fisher's distribution at a - **02** significance level of 05%

:We have the following two hypotheses

There is no relationship between all independent variables **:Null hypothesis** (X_i) and the dependent (Y_i) variable

$H_0 : \beta_1 = \beta_2 = 0$

There is a significant relationship between the **:Alternative hypothesis** independent variables (X_i) and the dependent variable (Y_i) i.e ,

$H_1 : \beta_1 \neq \beta_2 \neq 0$

:We calculate the tabular and calculated Fisher value as follows

that We know $F_{tab} = F_{01-\alpha,K-1,n-k} = F_{01-\alpha,V_1,V_2} \Rightarrow F_{tab} = F_{0.95,02,07} = 04,74$

Fisher's calculated value can be calculated in two different ways

$$F_{cal} = \frac{(n-k)\left(\hat{\beta}_1 \sum y_i x_{i1} + \hat{\beta}_2 \sum y_1 x_{i2}\right)}{(K-1)\left(\sum Y_i^2 - \hat{\beta}_1 \sum y_i x_{i1} - \hat{\beta}_2 \sum y_1 x_{i2}\right)} \Rightarrow F_{cal} = \frac{(07)(254,583676)}{(02)(01,816324)} = 490,57$$

permission $F_{cal} > F_{tab}$ We reject H_0 and accept H_1 that is, there is a significant relationship between the independent variables and the dependent variable (there is an effect of at least one of the two variables (X_{i1}) and (X_{i2}) the dependent variable (Y_i) (

:Analysis of variance table-

Source of variance	The sum of the squares of the deviations	Degree of freedom	Mean square error	FisherF value
The deviations shown by X_1 and X_2	$SSR = \sum(Y_i - \overline{Y})^2$ =254,583676	**02**	$MSR = \frac{SSR}{k-01}$ =127,291838	$F_{cal} = \frac{SSR}{SSE}$ $= \frac{127,291838}{0.259474}$ $= 490,57$
Deviations not shown	$SSE = \sum(Y_i - Y_i)^2 = \sum e_i^2$ $= 01,816324$	**07**	$MSE = \frac{SSE}{n-k}$ =0.259474	
Total deviations	$SST = \sum(Y_i - \overline{Y})^2 = \sum Y_i^2$ =256,40	**09**		

Testing the effect **-03** (X_1) individually at (Y_i) a significance level of 05% , .using an analysis of variance table

:We have the following two hypotheses

There is no relationship between the independent variable **:Null hypothesis** (X_1) and the dependent (Y_i) variable $H_0 : \beta_1 = 0$

There is a significant relationship between the **:Alternative hypothesis** independent variable (X_1) and the dependent variable (Y_i) i.e , $H_1 : \beta_1 \neq 0$

While we write the form containing the variable (X_2):as follows

$Y_i = \beta_0 + \beta_2 X_2 + \mu_i$

:We know that the estimating equation for the regression line is $Y_i = \hat{\beta}_0 + \hat{\beta}_2 X_2$

We extract the value $\hat{\beta}_0$ as $\hat{\beta}_2$:follows

$$\beta_2 = \frac{\sum y_i x_2}{\sum x_2^2} \Rightarrow \beta_2 = \frac{161,20}{117,60} = 01,37075$$

$$\beta_0 = \overline{Y} - \beta_2 \overline{X}_2 \Rightarrow \beta_0 = 11,60 - (01,37075)(09,80) = -01,83335$$

permission $Y_i = \hat{\beta}_0 + \hat{\beta}_2 X_2 \Rightarrow Y_i = -01,83335 + 01,37075 X_2$

The sum of the squares of the deviations resulting from the variable (X_2) is

$$SSR(x_2) = \beta_2 \sum y_1 x_2 \Rightarrow SSR(x_2) = 01,37075 \times 161,20 = 220,9649$$

The effect that the independent variable adds (X_1) :to the function is

$$SSR(x_1) = Y_i X^T B^T \text{-} \beta_2 \sum y_1 x_2 \Rightarrow SSR(x_1) = 254,583676 - 220,9649 = 33,618776$$

,To test the significance of this effect added by the variable (X_1) we prepare an analysis of variance table as follows

Source of variance	**The sum of the squares of the deviations**	**Degree of freedom**	**Mean square error**	**FisherF value**
The deviation described before X_2	$SSR = \beta_2 \sum y_1 x_{i2} \Rightarrow$ $= 220,9649$	**01**	$MSR = Y_i X^T B^T \text{-} \beta_2 \sum y_1 x_2 \Rightarrow$ =33,618776	$F_{cal} = \frac{SSR}{SSE}$ $= \frac{33,618776}{0.259474}$ =129,565
The deviation described before X_1	$SSR = Y_i X^T B^T \text{-} \beta_2 \sum y_1 x_2 \Rightarrow$ =33,618776	**01**		
The deviations shown by X_1 and X_2	$SSR = \sum (Y_i - \overline{Y})^2$ =254,583676	**02**		

Deviations not shown	$SSE=\sum(Y_i-Y_i)^2=\sum e_i^2$ $=01,816324$	**07**	$MSE=\frac{SSE}{n-k}$ =0.259474	
Total deviations	$SST=\sum(Y_i-\overline{Y})^2=\sum Y_i^2$ =256,40	**09**		

permission $F_{cal}>F_{tab}$ We reject H_0 and accept H_1 any (X_1) interpretation that affects us morally (Y_i)

Testing the effect **-04** (X_2) individually at (Y_i) a significance level of 05% , .using an analysis of variance table

:We have the following two hypotheses

There is no relationship between the independent variable **:Null hypothesis** (X_2) and the dependent (Y_i) variable $H_0:\beta_2=0$

There is a significant relationship between the **:Alternative hypothesis** independent variable (X_2) and the dependent variable (Y_i) i.e , $H_1:\beta_2\neq 0$

While we write the form containing the variable (X_1) :as follows

$$Y_i=\beta_0+\beta_1X_1+\mu_i$$

:We know that the estimating equation for the regression line is $Y_i=\hat{\beta}_0+\hat{\beta}_1X_1$

We extract the value $\hat{\beta}_0$ as $\hat{\beta}_1$:follows

$$\beta_1=\frac{\sum y_ix_1}{\sum x_1^2}\Rightarrow\beta_1=\frac{240}{226,50}=01,05960$$

$$\beta_0=\overline{Y}-\beta_1\overline{X}_1\Rightarrow\beta_0=11,60-(01,05960)(09,50)=01,5338$$

permission $Y_i=\hat{\beta}_0+\hat{\beta}_1X_1\Rightarrow Y_i=01,5338+01,05960X_1$

The sum of the squares of the deviations resulting from the variable (X_1) is

$$SSR(x_1)=\beta_1\sum y_1x_1\Rightarrow SSR(x_1)=01,05960\times 240=254,3046358$$

The effect that the independent variable adds (X_2) :to the function is

$$SSR(x_2)=Y_iX^TB^T\text{-}\beta_2\sum y_1x_2\Rightarrow SSR(x_2)=254,583676-254,3046358=0,279040238$$

,To test the significance of this effect added by the variable (X_2) we prepare an analysis of variance table as follows

Source of variance	**The sum of the squares of the deviations**	**Degree of freedom**	**Mean square error**	**FisherF value**

The deviation described before X_1	$SSR=\beta_1\sum y_i x_1 \Rightarrow$ $=254,3046358$	**01**	$MSR = Y_i X^T B^T - \beta_1 \sum y_1 x_1 \Rightarrow$ $=0,279040238$	$F_{cal}=\frac{SSR}{SSE}$ $=\frac{0,279040238}{0.259474}$ =01,07540
The deviation described before X_2	$SSR = Y_i X^T B^T - \beta_1 \sum y_1 x_1 \Rightarrow$ $=0,279040238$	**01**		
The deviations shown by X_1 and X_2	$SSR=\sum\left(Y_i-\overline{Y}\right)^2$ =254,583676	**02**		
Deviations not shown	$SSE=\sum\left(Y_i-Y_i\right)^2=\sum e_i^2$ $=01,816324$	**07**	$MSE=\frac{SSE}{n-k}$ =0.259474	
Total deviations	$SST=\sum\left(Y_i-\overline{Y}\right)^2=\sum Y_i^2$ =256,40	**09**		

permission $F_{cal} < F_{tab}$ We reject H_1 and accept H_0 that which (X_2) does not morally explain or affect (Y_i)

Chapter four

Chapter four ;

1. **Using the systems approach in preparing sustainable development plans:**

The systems approach or systems theory is a theory adopted in economic analysis, level, and is based on the idea that studying a specific whether at the macro or micro case must be by studying the parts or subsystems that make up this system and analyzing the existing relationships between them in order to know the extent of their .impact on the system

lies to the general cosmic system, which consists of subsystems The same app representing the ecological systems, which represent the environmental dimension, and humanity, which expresses the economic and social dimension. Therefore, the etween each must be known. Subsystems and the extent relationship and interaction b .to which these relationships influence the major balances of the universe

from

Popular participation, which means the participation of all societal groups and 2 the universe in order to approve all active institutions in society and in .development strategies

The World Bank only identified ten basic principles on which sustainable :[1] development is based

The first principle: carefully set priorities in order to focus on the most important ;points

principleMaking the most of every dollar by avoiding projects with high costs, : especially for developing countries that cannot afford it, and working to find the least .expensive ways to confront environmental problems

.to achieve profit for all parties The third principle: Seizing opportunities

;Principle Four: Use market tools wherever possible

.The fifth principle: Economy in using administrative and organizational capabilities

;Principle Six: Working with the private sector

;t of CitizensSeventh Principle: Full Involvemen

.The eighth principle: Employing partnerships that achieve success

The ninth principle: Improving administrative performance based on efficiency and .effectiveness

.Principle Ten: Integrating the environment from the beginning

:yChapter summar

Man has sought from the beginning to achieve and satisfy his needs and improve his being. This pursuit has led him over the course of experiences to -levels of well formulate many theories and draw theoretical and applied frameworks for them. With many concepts and terminology have emerged, the most prominent of which ,them economic growth, economic development, -are what our study dealt with development. Humanity and sustainable development are approaches whose purpose e ways and means that guarantee a person access to is to search for the most effectiv

[1] Khababa Abdullah, "Sustainable development principles and implementation from the Rio de - intervention at the international forum on Janeiro conference to the Bali conference 2007," in an sustainable development and the efficient use of available resources, Farhat Abbas University of Setif, .April 7, 8, 2008

being. The most -his basic goals and advance to the point of achieving absolute well :important results of the first chapter can be summarized as follows

- t be completely The concept of growth and economic development canno separated, as the two concepts are related and interrelated. Developments in the economic environment have led to the expansion of the boundaries of economic growth, so that the concept becomes larger and more comprehensive, .ic developmentwhich is econom
- The Islamic approach is considered an integrated approach as it addressed all the basic issues in order to achieve the development and architecture of the earth, some of which were later discovered by positivist thought, and many of are still being discovered today and whose validity is proven practically which .and experimentally
- When studying any development model, we must take into account the circumstances in which it prevailed and understand that its success in certain .ot necessarily mean its success in other countriescountries does n
- and this may be -Modern economic thought got rid of excessive materialism to focus on issues of another kind, such as protecting the -only apparent e with the principles environment and preserving its components in accordanc .of sustainable development

The concept of sustainable development is a concept that has many branches and overlaps. It has raised many issues The same common interest among countries that formulas for joint cooperation that can be have tried and are still trying to search for .preserved Everyone's interests

In order to evaluate the paths of various countries and measure achievements in light of the requirements of sustainability, it was necessary to find means to measure makers in drawing up national strategies by giving them -assist decision progress and a true picture that reflects reality at all levels. These means are known as indicators of .sustainable development. Which we will devote the second chapter to studying

Chapter II:

ethodologies for measuring growth, economic development and sustainable M development between theory and application

Chapter four ;

: Introduction

Sustainable development has established a new approach and form of achieving t many goals, the most prominent comprehensive economic development, and has se of which are meeting the requirements of the present, addressing the negative effects left by past experiences, aspiring to preserve the rights of future generations and depth -iples and objectives: Inprotecting the natural environment. These needs Princ studies to analyze reality and anticipate the future. This is why the necessity of measuring sustainability has emerged. Thus, the topic of preparing comprehensive mportant topic in indicators to measure sustainable development has become an i which many studies and efforts have been focused. During this chapter, we will try to study the most important points related to measuring sustainable development, :according to the following plan

methodological framework for studying The terminological and **:The first section** and analyzing indicators for measuring economic growth, economic development and .sustainable development

Indicators for measuring economic growth and economic **:The second section** t of view to the traditional point of view development from the traditional poin .Modern

Sustainable development indicators: The United Nations vision **:The third topic** .towards an integrated model for measuring sustainable development

Chapter four ;

framework for The first section: The terminological and methodological studying and analyzing indicators of measuring economic growth. Economic development and sustainable development

Before delving into the study of indicators of economic development and sustainable concepts that define the framework of the development, we must clarify some of the study, in order to understand them well and thus avoid falling into terminological fallacies, by clarifying the concept of the indicator and how to prepare it. We will hes and methodologies adopted in also try to address the most important approac .Preparing sustainable development indicators and methods for engineering them

It is true that measuring the growth and economic development of countries is not a he beginning of the recent demand, but rather it appeared since the 1940s with t emergence of the concept of national accounting. Since then, the gross national product has become the basis of measurement, as its values and changes have come -c wellto express the extent to which countries have achieved progress and economi being or not. Until the year 1992, the year of the Rio de Janeiro Summit, which was the beginning of an important phase that called for the search for alternative that -more generally economic indicators -indicators of gross domestic product be a base of quantitative information that would help measure the steps and would paths followed and assess the extent of their direction towards achieving... .Sustainability

The first efforts of this summit were translated into a document known as Agenda 21, he last chapter of which (Chapter Forty) focused on the necessity of improving t previous information systems and developing other new systems that would help in making decisions at different levels, and the next paragraph of Chapter Forty explains :that

e use of some indicators, such as gross product, per capita income, or even hT“ pollution levels, is no longer useful in order to measure sustainability, and the methods used to estimate and calculate the relationship between the various sectors ntrolling development, such as the environment and demographic and and aspects co social variables, are not developed and are not even applied. Therefore, development indicators Sustainable development must be developed in order to be a strong making at all levels and contributes to creating -decision information base that helps regulation that integrates the environmental aspect into the -sustainable self development system. ”

:The United Nations, through the previous paragraph, called for the necessity of

velop indicators to measure sustainable development. This call is Develop and de 1 governmental organizations in -directed to all countries and governmental and non order to develop a set of indicators as well as encourage countries to use new ;g and modify their traditional systemaccounts in national accountin

.Promoting the use of sustainable development indicators .2

The same paragraph adds that “States and international organizations at all local, nformation national and international levels must review and strengthen systems and i services in the various sectors related to sustainable development, and in particular -must ensure that the information provided is effective and usable for decision we must making, and that it is It is also directed to different categories of users, and work to strengthen existing mechanisms and strive to find new mechanisms that economic assessments to be reconciled with the final -allow scientific and socio information that is used in planning. ”

ome official governmental Since then, many attempts have emerged made by s governmental organizations with the aim of -organizations, research centers and non preparing and defining a set of scientific and practical indicators to measure alternative sustainable development. We will try to find out the most important indicators to the GDP index, and we will focus on the indicators developed by the .United Nations. As an integrated model for measuring sustainable development

The first requirement: indicators

measuring performance and The indicator is considered an effective tool in describing reality. In order to understand more the importance of the indicator, we will try to investigate its nature in general, and then try to identify the most important ndicators for measuring methods and approaches that were adopted in preparing i .sustainable development

The first section: the concept of the indicator and its characteristics

The indicator is used to determine results, evaluate performance, and translate study and reliance in the process qualitative results into numbers that facilitate their of making important decisions and developing policies and strategies. However, its use is subject to many restrictions and is hampered by many theoretical and aration or at the level of operational limits, whether at the level of definition and prep . .translation and analysis

First: the concept of the indicator

According to the writer Rashad Ahmed Abdel Latif, the word “indicator” means “in lects, the linguistic concept what indicates or clarifies something, and in practice it ref directly or indirectly, quantities that are not directly measurable or directly observed. ”

An indicator is a characteristic estimated by statistical methodologies or determined by calculation, allowing us vement, By identifying a positive or negative impro whether quantitative or qualitative, in the behavior of a variable attached to it, so the indicator is not It is a simple collection of data and is not just a statistical value, but .the studied reality rather a value that must have expressive power A representation of The indicator is quantitative information that allows describing the development of a specific situation, phenomenon, or activity and determining its results, evaluating and ive, that is, in a comparing them over time. The indicator can also be descript qualitative form. We also call by the indicator what was a fixed quantity at a specific moment or period, in relation to To a specific scale specific to the situation or s usually quantitative, condition, or to the effect of a specific variable. This variable i .recorded and can be tracked. And analysis

variable that expresses an -concrete -Simply put, the indicator is a tangible .intangible fact

Chapter four ;

The index has been used from the beginning in topics of a purely scientific nature, It is the translation of theoretical concepts into existing concrete was its goal and this variables that allow us to experiment with scientific theories. This is whatLazarsfeld expressed as “making social theories operational,” where concepts were first .and transformed into clear indicators and evidence analyzed

Indicators are usually used to study the evolution of the behavior of a particular phenomenon over time, or to compare a group of phenomena over a period of time, :must be characterized by either fixed or variable. The indicator

- ;Continuous followability over time

; via Place Followability

;calculability

- ;Simplified analysis and understanding by all users
- Easy to formulate
- .Ability to make local and international comparisons

:be an analytical tool, it must be allowed for us In order for the indicator to

;Going back to the past -

;Continuous daily management in the short term -

.Developing plans for the medium and long term based on forecasting from the past -

the phenomenon under study. The The nature of the indicator is determined by indicator may be one variable or... A function of several variables, and these variables are usually of a quantitative nature, but the matter is not without the presence of .variables of a qualitative nature

translation -quantitative -We can say that the indicator is the digital Than Previously of a set of data The treatment of a particular phenomenon, which is usually characterized by ambiguity and complexity, is transformed into simple, brief, and on that can be measured and analysed, which constitutes an interconnected informati information base that expresses the phenomenon under study. It will help in making .decisions, evaluating them, correcting them and amending them when necessary

et indicators by simply identifying the phenomenon It is not possible to identify and s studied. Rather, it requires some stages, whichLazarsfeld summarized :in the steps

:A brief explanation of the scheme is given as follows

various The first step: moving from the concept to the dimensions for the The first stage is concerned with the precise definition and specification **; dimensions** of the various dimensions that make up the phenomenon under study, by dividing it identifying the into a group of partial phenomena that make up it, with the aim of .points that will be studied later

where the previous **,The second step: moving from dimensions to indicators** dimensions are divided and analyzed into a group of Variables that are later of setting and identifying considered indicators of measurement, and the process depth theoretical study of the studied phenomenon that -variables depends on an in .allows knowledge of the factors that control it

After defining the **. The third step: Moving from indicators to measurement** easurement stage, where the spatial and temporal scope of indicators comes the m measurement is determined, as well as the units of measurement, especially for indicators that use -aggregate indicators, which consist of a group of different sub fourth step : Moving from measurement to multiple units of measurement. The indicators and measuring them experimentally, we -evidence. After defining the sub may find that they alone do not express the behavior of the studied phenomenon. cators must be collected and Therefore, the results and values of the partial indi .presented in a single value that expresses the overall value of the evidence

Second: Levels of aggregation and types of indicators

The goal of developing indicators is to reduce the number of preliminary data that when trying Analyzing or studying it in its raw form to make it may be lost meaningful and effective, this simplification and compilation of information is done to varying degrees and at multiple levels to form what is known as composite ample, the gross national product index is expressed in indicators or evidence. For ex financial totals of certain amounts, which represents a simple and traditional level of aggregation as It collects variables of a homogeneous nature (monetary units) and conomic dimension of development. On the other addresses one dimension, the e hand, if we take theIDH Human Development Index we find that it combines... , Between three indicators linked to three aspects of development, which are lifelong These indicators are of a different .knowledge and indicators that express income nature and use different units of measurement, which requires compatibility between .indicators that make up the guide-these sub

أ- **?What do we mean by aggregation**

Aggregation isthe data into information process of condensing information or raw One, where the indicators that receive greater weights are identified and determined according to importance, and the nature of the relationship between the indicators that ultiplication or from are collected in one guide is determined, is it. Addition, m .relationship another type

How it is done ?Preparing aggregate indicators for sustainable development

As we mentioned previously, the preparation of aggregate indicators requires the enting the various dimensions of integration of a set of secondary indicators repres sustainable development. In order to compare and combine the distinct elements (the growth rate of the gross domestic product, the emission of greenhouse gases, and turned to a common scale of hope in life, for example), the variables must first be re measurement, and the principle is Evaluating each indicator in relation to a scale and -range of change observed over time or defined in a nominal way, so that each sub ulated and collected indicator or variable is followed up and its values are calc .according to their weights to finally give the final value of the aggregate indicator

Preparing and using the composite indicator or index is hampered by an important factor, which is determining the specific weights for each partial indicator that is included in the calculation of the aggregate index. It is difficult to distribute the hts between the partial indicators, and to determine which of them receives a weig depth study through which the -greater weight. Therefore, the matter requires an in impact of each is determined. A partial variable in the overall behavior of the .menonexpressed pheno

ب- **:Types of indicators**

The nature of the indicators varies depending on the phenomenon studied and the ...phenomena that constitute it, and it can-sub Indicators may be either in unrelated, .composite or representative form

It is used to analyze phenomena into a group of **: Uncorrelated indicators .1** elements or factors and then choose a set of indicators to highlight them, that is, deconstructing the phenomenon into unrelated variables and treating each .independently -variable - phenomenon. Partial

In this case, the indicators are built on the basis of **: Composite indicators .2** adopting the largest possible number of factors that express a number of phenomena. ance and then Special weights are determined for each topic according to import .combined. To have a meaning

The most representative indicators and the most **:Representative indicators 3** interconnected indicators are selected, then the best indicators are weighted in terms all variables related to the of treating the studied phenomenon, that is, not phenomenon are taken into account, but rather they are represented in other variables that are able to express them implicitly. There are those who generally divide :indicators into five types

- ;llow us to monitorActivity indicators (processes): a
- .Results indicators: show us the results of the completed operations
- ;Performance indicators that allow measuring progress in achievement
- Comparison indicators: allow for comparing performance, whether internal or .external
- .measurement indicators Satisfaction

Section Two: Characteristics of indicators for measuring economic development and sustainable development and their importance

In order for an economic or social variable to **: First: Characteristics of indicators** opment indicator," it must: Representing some of the factors that be called a "devel shape a development process or state, the indicator can constitute a direct, complete and ad hoc measurement Of development, it is thus a development indicator, given ures is a development goal or one of its elements. When this that the aspect it meas goal or element is not measurable in itself, the indicator primarily serves to indicate as best as possible this goal or element. For example, the child mortality index .To measure the level of public health .constitutes an indicator

:Through this definition, we find that the development indicator must

- expresses a -which the indicator measures -The variable component development goal or one of its components Here, the variable is considered the .target

- Must in case it is not possible Through it, the quantitative expression of the .target variable is one of its elements It contributes to defining and measuring it

According toMpeqb :the sustainable development index should be ,

- ;issue or topic to be studied Relevant to the
- ;Sensitive to change over time
- Sensitive to change via Place
- ;Dynamic and able to measure progress in a particular field
- ;Repeated, comparable and based on regularly collected data
- ;Real and reflects reality
- .information Provides timely

Indicators are used to achieve many goals, **: Second: The importance of indicators** :including

- ;Describe and identify specific development situations
- ;Compare trends in society with development goals
- ;ariablesAnalyzing relationships between development v
- ;Predicting changes that may occur in society
- .Consolidation General and subsidiary goals of development

As for sustainable development indicators, they help achieve a set of necessary :requirements, which we will mention Of which

- s, developing and reviewing national strategies as well as Making decision making -public policies because they constitute a leadership board for decision .at various levels
- ;Setting goals and following them
- es, economic It allows us to make comparisons between different units (countri ;(...institutions, administrations
- ;Identify, evaluate and allocate resources more rationally
- .Informing the public and various stakeholders

may encounter the process of preparing and selecting measurement indicators, development measurement indicators Sustainable many especially with regard to Challenges, especially those related to the methods of adopting some of them, as the number of indicators may reach The indicator used to measure sustainable threshold, and because it is... It is 100 development sometimes reaches or exceeds the difficult to use all of these indicators, so a limited number of them must be chosen, :[1] and for this we need... A set of selection criteria, the most prominent of which are

1 -

1. ;quality of information
2. ;information Access to
3. ;Measuring goals
4. ;Applicability and flexibility to changes
5. ;Simplicity and ease of understanding
6. ;Comparability in time and place
7. .Facilitating decision making and assigning responsibilities

criteria, as the selection of The first three criteria are considered preparation indicators depends on the degree of their verification, because it is not possible to prepare or select an indicator without the presence of a base of correct and continuous ees the validity and continuity of information (past, present, and future) that guarant measurement. As for the remaining criteria, they are criteria that govern the measurement. The quality of the indicator after use, and we may add a very important to reflect the priorities criterion, which is the ability of the selected indicators according to the strategic goals established within the framework of sustainable development. In my opinion, this criterion remains the most influential on the inable effectiveness of the indicators chosen to represent and measure susta .development

The second requirement: the approaches adopted in preparing sustainable development indicators

There are many approaches adopted in preparing sustainable development indicators, try to choose a group of them, and because it is difficult Study it All of them, we will as a result of many reasons, the most important of which is the availability of informationPhilippe DEFEYT rehcraeseR gniraperp rof ygolodohtem deifilpmis A“ : basic axes Philip presented sustainable development indicators based on three

According to this axis, sustainable development indicators look at: creating **:Vision .1** ;new indicators that improve the performance of traditional gross domestic product

without focusing Creating indicators that can study all aspects and angles of society * .on the economic aspect

which determine the basic topics that must be focused on in **:Basic concerns .2** :preparing indicators Sustainable development, which is

; Concerns (social issues *

.Environmental concerns *

1: Comparative analysis of durable development systems, OP.cit

: Philippe DEFEYT, "Le social et l'environnement: indicators alternatifs au PIB", Institute for Durable Development.

http://www.ulb.ac/students/desge/courses/envif443:COUR08-4.indic 07/28/2010 s adopted in preparing sustainable development method **Methodology : The** indicators and are in the form of: a dashboard that collects a number of indicators with the aim of giving a picture of consensus between all aspects of development. ;sustainable

.Aggregate indicators

e manyThere ar

Among the philosophies from which efforts to define and measure sustainable development begin are: It has many scientific approaches based on the process of preparing sustainable development indicators, and among the most important These .will be discussed below approaches

The first section: Introduction to capital

there Who considers sustainable development to be a historical extension of previous theories and models in economics? Which was based in its entirety on the concept of to achieve and measure it in society, and the majority agreed. being and ways-well Researchers and scholars believe that sustainable development aims to determine the -being of current generations and the well-existing relationship between the well This relationship has been explained by the overlap .being of future generations being of the individual, especially capital. -between the concept of capital and the well Natural money and ways to exploit and harness it in the present without prejudice to .ons to use itthe right of future generati

Capital means the available stock of any commodity or material, regardless of its nature, and used in production operations for a specific period until it runs out. h available Through this approach, the various types of capital that represent the wealt to countries are evaluated and measured in order to determine the levels of consumption and substitution among them in a way that ensures the extension of wealth. Luxury from! Current generations to future generations, and the use of this ach in measuring sustainable development is linked to the necessity of appro being cannot be sustainable -achieving a condition, which is that achieving well unless we replace or preserve resources and wealth with their various components for orever. The wealth of countries is usually divided into five Types many years, if not f :[1] of capital, which are as follows

.such as stocks, bonds, and cash deposits **:Financial capital**

building equipment, communications equipment and various **:Productive capital** types of infrastructure. Natural capital includes the stock of natural resources, land and ecosystems. Human capital: This concept is considered relatively new. This term was first used byGary Beck andSchultze in the 1960s. Human capital expresses a country's stock of individuals with competencies and knowledge who are able to

[1] -1: Measuring Sustainable Development, 2007, op.cit .

being of -produce and thus contribute to achieving the economic and social well . .individuals

1: Tracey Strange, Anne Bayley , “The sustainable development of economic development, society and environment”, OCDE. www.ocde.com

Social capital is represented by social networks, institutions, etc It is created by the .compatibility of the fabric Social

countries must strive to increase the stock of each type ,And just With this approach of capital, or at least keep it constant, in order to meet immediate and future needs. The concept of sustainability, according to this approach, is linked to the extent of the uses of available resources, and according to this perspective, it ability to rationalize has been considered The United Nations defines sustainable development as development that ensures that per capita income does not decline of national wealth sources of this wealth, which are represented by stocks by replacing or preserving the All of productive capital, human, social and natural, and the relationship that exists :between the five types can be expressed as follows

TNW = pF* + pR + pN + pH + pS

:ressesWhere each of the elements exp

TNW total national wealth :

F , financial capital :R: ,productive capitalN ,natural capital :H ,capital human :

S .social capital :

P The accounting price (value) for each type of capital, which represents the change : being as a result of the change in the marginal value of each type -in the level of well .of capital

ding to Progress towards sustainable development is evaluated and measured accor term monitoring of indicators that represent the various wealth -this approach by long controlled by the state, which focuses on measuring the stock of various national .(capitals (natural, human, productive, social and financial

:Section TwoDPSIR Response Model-State-Pressure

This model was initially used to evaluate the state of the environment and its living systems, and is distinguished by the fact that it simultaneously identifies the causes of ture of the solutions proposed to problems and their consequences, as well as the na " treat them. The wordDPSIR in English is derived from the first letters of the " following words: Drivingforce Pressure Pressure State Impact ResponseResponse .

pressure, state and response, The model initially consisted of three elements, namely and it developed to include the other two elements. The idea of this model can be :summarized through the following representation

Response, decisions and corrective actions, policies, legislation, regulations and agementresource man

The principle of this model is that the pressing forces that express, in their entirety, the various human activities, industrial sectors, consumption, technology, and the population growth, constitute great pressure, especially on the environment and various systems that compose it. These increasing pressures generate negative effects at various levels, such as the impact on the health of living organisms and resource his systems in various ways. Its types and the resulting negative economic impacts. T abnormal situation requires finding solutions within different frameworks, whether economic, legal or/and regulatory. This approach is widely used by countries and international organizations such as the United Nations in preparing sustainable .ent indicatorsdevelopm

(Branch Third : Dimensional approach (sectoral approach

This approach is considered the most widespread among the existing approaches. It has been adopted by many countries, international organizations and institutions due of its use. This approach divides sustainable development into to the simplicity dimensions that are independent of each other, which are the economic, social and .environmental dimensions, in addition to the institutional dimension in some cases

advantages of this approach is that it allows asking most of One of the most important the questions related to the concept of sustainable development and also provides us with a balanced representation of the dimensions of sustainable development. nd, the absence of interconnection and overlap However, we note, on the other ha between these various dimensions, and the process of selecting indicators and classifying them according to the main dimensions is subject to subjective and e to include one indicator within two personal factors. In addition to this, it is possibl or more different dimensions, 1 especially for phenomena that can be described from different economic, social, and environmental angles. This engineering enables us to up sustainable development and the identify the most important dimensions that make topics that branch out from each dimension, but at the same time it may not give us an integrated picture of sustainable development among them, nor does it define for .us the goals and priorities to be achieved

being-our: Introduction to wellSection F

This introduction has focused on the interrelationship between sustainable being-development and the ability to achieve well

Sustainable, according to Dasqita And Samuelson " And Dasgupta that the " being In reality, it depends on the extent to which -ellreflection of economic w individuals enjoy and acquire their rights to exploit and enjoy the distributed resources according to what They expect and what they aspire to, that is, the concept cept of wealth and the methods of distributing it being is linked to the con-of well among individuals in one society and among future generations. There are those who define sustainable development according to this perspective as the continuous eing forever or at least for a very long b-increase and development in levels of well period of time, i.e. Sustainability is measured by the extent to which individuals .being over time-achieve well

Chapter four ;

Section Five: Introduction to objectives

is to identify and This approach works according to a simple methodology, which prepare sustainable development indicators. For strategic goals and priorities that being of its citizens, and so it is It -require state intervention with regard to the well adopted and the ensures compatibility and harmony between the general policies national goals determined by the strategy

: Comparative analysis of durable development systems, op. cit .

Measuring Sustainable Development, 2007, op.cit .

wolla srotacidni ,hcaorppa siht ot gnidroccA ".tnempoleveD elbaniatsuS lanoitaN" for comparative evaluation of the work and progress achieved With the objectives .specified in the national strategy

This method allows us to intensify and focus efforts on priority points Indicators strategies within the allow us to follow up on the implementation of national framework of sustainable development. The United Nations has called and requested countries to prepare their national strategies for sustainable development and support .them with a set of indicators

ieldsSection Six: Introduction to the f

The fields of sustainable development express the fields and areas of study, and this approach allows us to define Understanding the stakes at all levels and making o various decisions in light of sustainable development, engineering goals are linked t .the basic fields except in some exceptional cases

The previous approaches are the most widely used methods in engineering sustainable development indicators in the world, and the use of these approaches and Rather, we find that in many .methods in many cases is not done separately experiments more than one method and approach has been relied upon to prepare sustainable development indicators. In order for the selected set of indicators to rement of the ensure a more comprehensive representation and more accurate measu .progress achieved in the sustainability path

What can be noted in our presentation of the most important approaches used in preparing sustainable development indicators is that it is not possible to choose to prefer between them except through a group of between the previous approaches or objective factors, because each approach was built on a philosophy whose assumptions differ from the others, and the situation is specific to each country. It is not possible, for example, to choose plays a role in choosing a specific approach. It the capital input if there is no real ability to identify and evaluate all currently available stocks, as well as determine current and future consumption rates, buted over different fields as well as especially since this requires a lot of effort distri in continuous stages. It is also not possible to The goals approach was chosen by a country that had not originally determined its strategy or national goals for some positive and negative sustainable development, and the following table shows .points in each of the previous entries

Chapter four ;

1: A preliminary list of indications for ongoing development to surveil and monitor the progress of the project in Québec and the ongoing development maturation, June 12, 2009, page 20.

Positive and negative points in approaches to preparing sustainable (1-le (2Tab development indicators

Negative points	Positive points	methodology
doesn't happen Work on the interactions between The basic axes of sustainable .development	Dividing sustainable development into specific work axes and divided into Social, ,axis economic environmental and .institutional Ease of application and work according to this .curriculum	Dimensions entry
Difficulty in evaluating some types of capital, such as human, social, and often .natural capital Difficulty in finding appropriate substitution methods between the five .apitaltypes of c	Optimal exploitation of meet capital in order to .current and future needs Determine the stock of each .type of capital	entrance capital
It is difficult to determine the nature of each variable or phenomenon, only the phenomenon is in the form of pressure and impact at the .same time	Determines the nature of each variable and its impact .on achieving sustainability Determines the interaction .between variables	model DPSIR
The difficulty of finding and defining frameworks, whether at the conceptual level or at the applied level, being, -for the term well which makes it difficult to .measure its level	Linking the concept of sustainable development to the first goal of development of all types and with its multiple approaches and names, -which is achieving well being Improving the living of the standard of .individual	entrance being-Well
Neglecting some points that	Determine priority strategic	Introduction

may not be included among the priorities but have an impact on the sustainability .process The difficulty of accurately determining the temporal and spatial framework of the ally for targets target, especi .that are difficult to quantify	goals within the framework .of sustainable development Allocating resources according to priorities, which leads to efficient use .and effectiveness in work Defining the responsibility of each party facilitates the ompletion, process of c .up and investigation-follow	to goals

Source: Prepared by the student

The third requirement: The problems associated with engineering sustainable development indicators and the practical solutions proposed for them

ustainable Development of the Ministry of Sustainable The Coordinating Office for S Development, Environment and Farms in the Province of Quebec conducted a six models of sustainable development indicators, which -comparative study of thirty tions and problems that hinder the process allowed the identification of a set of limita of preparing and designing sustainable development indicators. The office's report :[1] summarized them in two basic problems

Responding to the sustainable development goals. Despite the different and multiple - methods used to engineer indicators, we find that they generally focus on a fundamental point in the process of selecting indicators, which is their ability to respond to sustainability challenges at all levels and reflect the goals of sustainable .ent in an appropriate waydevelopm

Overcoming ambiguity in terminological and methodological terms when preparing - and selecting sustainable development indicators, especially with regard to defining ap between the concept the axes, topics and indicators. In many cases we find an overl of field , dimension, and topic , and the classification of indicators according to the dimensions of sustainable development is subject in some cases to personal and .subjective estimates

ion Office for Sustainable Development In the same study conducted by the Coordinat of the Ministry of Sustainable Development, Environment and Barns in KBA Province , and based on the models included in the study, solutions were proposed to :hich areconfront the two previous problems, the most important of w

Peaceful hierarchical presentation of indicators, as many countries decided to present development indicators Sustainable sustainability at different levels. This is to avoid indicators reflect using a large number of indicators presented in one set, so that these priorities and to facilitate their study and understanding. There are many experiences that It was presented according to this method, including, but not limited to, the uropean model presented by the European Union following its review of the E

1 -

Strategy for Sustainable Development in 2006, where it presented a set of indicators. It is divided into three levels, including twelve key indicators located at the top of the d to strategy up of political goals and are directe-pyramid that allow for the follow four -officials, decision makers and the public. The second level contains fifty indicators related to priority fields of activities, and the third level includes ninety It tracks the depth analysis of policies and understanding-indicators that allow for in evolution and complexity associated with sustainable development. Defining and quantifying goals in order to focus the efforts being made. One of the most prominent attempts in this regard is the experience of the "Japan for Sustainability" ,organizationwhich prepared to measure sustainable development -twenty indicators classified according to the dimensions of sustainable development, and a -in Japan value was given An initial absolute value for each of the indicators and calculating its percentage compared to the value that was specified as the intended target

1: Comparative analysis of durable development systems, op.cit.

2: ibid.

Achieved by the horizon (2050) and followed up over time at the local and national .d percent is reached It means achieving set goalslevels until one hundre

second section : Indicators for measuring growth and economic development: moving from the traditional view to the traditional view Modern

development Before delving into the discussion and analysis of sustainable indicators, we see that: It is necessary to get to know the traditional indicators that first appeared to measure growth and economic development, and we will divide this measuring section in which we will first learn about the traditional indicators for economic development, and secondly about the most important alternative indicators that came in the same context as the first, but in a corrected and modified form for the to indicators old indicators such as gross product, and then The study will be devoted .that carried a new and different perspective in terms of concept and methodology

Many economic indicators have been used to measure economic growth and economic development. Economic indicators are defined as "indicators that describe aracteristics of the country's economic and social system. They can be the ch presented in the form of an average rate from a total mass, such as the annual per) capita income, or in the form of percentages of the gross national productGNP ,(f export, import, or debt, or presented in the form of ratios." Among such as the rate o them, such as debt service compared to the value of exports, and the most prominent of these indicators is the national product, the total gross domestic product, or per .capita

first requirement: indicators for measuring economic growth and economic The development

Indicators for measuring growth and economic development have moved from traditional indicators concerned with the material aspect only to more open hey remain enclosed under the banner of the materialistic view indicators, although t of the concept of growth and development and the new changes. We will first discuss .the indicators

economically, but they tried to adapt. Sentence with. from

.conomic indicatorsTraditional economic then modern e

The first section: traditional economic indicators, which depend in general on .income, its distributions and composition

:Among the most prominent of these indicators we find

:First: Raw domestic product

in the gross internal product (gross) is the main Measuring and tracking changes indicator for measuring economic activity in any country. When we say that the growth rate of the economy in a country has reached a certain percentage, we are .ross domestic product in this countrymeasuring The real annual development of the g

The raw internal product represents the final outcome of the production process of the production units established within

Homeland, with the end of World War II included the list of products that are he internal product Raw material goods and, to a lesser extent, only counted within t marketed services (services provided by public -services that can be marketed. Non administrations) were not included in the calculation of the gross domestic product .76except in the year 19

The internal product is considered one of the most famous indicators for measuring economic development and one of the most widely traded and used to express the economic situation of countries. It can be calculated according to one of the ree methods: the sum of the final uses of goods and services by economic following th agents residing in the country (real final consumption plus net capital formation). .Fixed) In addition to exports and subtract imports of goods and services)

imports -on = goods and services consumed + exports Internal (raw) producti

The total wages received by employees and workers, surpluses from export - .operations, and taxes related to export and import

ties. This The sum of the net added values of the various sectors and economic activi - method allows avoiding double counting of goods that can be considered goods for final consumption and for intermediate consumption at the same time because it considers that the production of goods and services intended for final consumption quires the use of intermediate goods and therefore the difference between the sale re or exchange price and the value of the goods. The input used in production expresses the added value, and from here we find that the raw internal product expresses the .f the added values of all activities Economicsum o

Although the gross domestic product represents the mainstay of national accounting and the most important and most widely used analytical economic indicators, it has cism about its ability to give a true picture been subjected to much criticism and skepti of the levels of growth and development. The most important drawback known to income indicators is their focus on the material aspect of calculation and estimation at the indicator "measures the increase in and their neglect. The intangible aspect is th output regardless of whether it is due to a real and continuous increase in society's ability to produce, and to a real and permanent expansion in the opportunities rds of living, or whether it is due to available to its members to improve their standa contingent circumstances such as fluctuations in international trade or other causes." Deterministic, such as petroleum or mining discoveries or changes in... weather .[1] condition

1: www.insee.fr le 10/05/2010

: What is the economical cruise? indicators and methods :

www.cesa.air.defense.gouv.frIMGpdfMesurer_la_ croissance_economique.pdf 12/07/2010

.Issawi, previous reference, p. 102-Ibrahim Al :3

this indicator can The most important and prominent limitations and shortcomings of :be summarized as

- ;It does not measure quality of life
- ;It does not reflect how wealth is distributed
- .Eliminates environmental and social considerations in measurement

Second: Per capita gross product

national product expresses the result of dividing the The per capita share of the gross output or income by the population in a given year. It expresses the share of the value of goods and services produced within a country during the year per capita. Many ones, are interested in following the development countries, especially developing path of this indicator to verify its ability to reflect their development efforts on the individual. This indicator, in turn, has been subject to discussion about a set of points ount in analyzing developments in the average per capita that must be taken into acc share of national product without being satisfied with its growth rate. Increasing its :value, among these points we mention the following

- e national product The method of calculation, where the question remains: is th divided by the total population or only by the active group that receives income on a regular basis, because it is illogical to calculate the groups? from without ;income
- way the latter The per capita share of the national product does not reflect the is distributed among the various social groups and classes. It does not reflect the degree of justice in the distribution of wealth, as it provides us with a fixed ;average value that cancels out the values. maximum and minimum
- a share of the national product does not reflect the purchasing The per capit power of income, which varies between segments of society, but rather it is

[1] -

always the same, as we find in fact that an increase of a certain amount in the y Means Doubling purchasing power, average income of the groups Poor ma while the same increase in the average income of the rich groups does not produce the same effect, and this is based on the analysis of marginal utility. fixed We also find, on the other hand, that an increase by a certain and percentage in the average income of all groups may enhance the imbalance in the distribution of wealth, as the percentage 5% As for the income of the rich groups, it is not equal to the same increase as that of the poor groups in real .monetary terms

- Taking the per capita GDP index as a tool for international comparison is subject to conditions And the adjustments are a result of the difference in the .purchasing power of money from one country to another

:domestic product Third: The ratio of total investment to gross

This indicator refers to spending on additions to fixed assets to the economy as a percentage of Gross Domestic Product: This indicator measures the ratio of It .investment to production, and reflects the general trend of countries' spending allows knowing the quality and nature of investments, which reflects the broad .outlines of the strategy adopted by the state

(Fourth: Total external debt as a percentage of gross national product (raw

ndebtedness and helps assess their This indicator measures the degree of countries' i ability to sustain debts The debt to resource base indicator shows the extent to which a country is able to transfer resources to export production with the aim of enhancing .[1] The ability to enhance

Fifth: Inflation rate

There are many types of inflation, but talk about this phenomenon is usually linked to the continuous rise in the general level of prices during a certain period of time, .[2] which is accompanied by a decrease in the purchasing power of consumers is usually measured by calculating the relative change in the price index for Inflation) consumptionprix à la comsommation) as an increase in this index reflects the , increase in the costs of life, and this measures The indicator is price Basket of ducts during a certain period depending on a specific reference year, necessary pro :and the inflation rate is calculated according to the following equation

DA for year (N) / DA for year N * 100 -Inflation rate (DA for year 10

D E: “ IPC ediug ecirp remusnoc eht si ”

The previous group represents a sample of traditional economic indicators that are used to measure growth rates and economic development in a country and the quality and trends of this development. Income indicators reflect to us the general trend of onal income between rise and fall. This means an increase or decrease in the nati added values that have been achieved at the level of all activities. Practice within a

[1] -

[2] -

year also shows us the ways to distribute these added values among individuals, ment or consumption indicators show us the general behavior of the two while invest .phenomena and their effects on growth and economic development

.Muhammad Tahir, Qadri , previous reference, p. 170 :

: http / /:www.colvir.netproflouise.bouffardeco-31onotesinflatio-notes.pdf

Section Two: Alternatives for measuring economic growth and economic development according to a more comprehensive view

This group contains a number of indicators used to measure sustainable growth that are considered alternatives

.income indicators and traditional indicators Complementary to

First: the adjusted raw internalproduct

Since the 1960s, some American economists have been working on reformulating being according to a new perspective, and thus -indicators that express economic well many indicators have emerged. Alternative, such as modified gross domestic product. oncept expresses the modification and correction of gross product through This new c public and private expenditures within the framework of protecting the environment It includes the gross domestic product . .[1] and combating pollution

new -lso known as the sustainable internal product or what is a -The adjusted GDP calculations that include the costs of damages resulting from pollution, which mainly include the costs of preventing expected environmental damage and the costs losses, meaning that the adjusted GDP has resulting from actual (real) environmental maintained the same approaches. The traditional GDP calculation only added some .calculations that reflect the costs of protection and treatment the environment

Second: IndiceDe Bien-être Economique

being goes to -reating this new philosophy in calculating economic wellCredit for c the work ofDalhousie University where this researcher devoted himself to studying , eighties of the last century. In 1998, in -being in Canada during the mid-well cooperation with the researcher from the CenterFor The Study Of Living Standards, he developed a set of economic in From"Andrew SharpeIndicators to measure being in Canada. In 1999, the study was applied to the United States -economic well of America. In 2000, the two researchers applied these statistical indicators to six) countries of the Organization for Economic DevelopmentOCDE This work .(quickly turned into a global reference. It was seen as a genuine attempt at a new and being. This evidence is based on -different account of the concept of economic well mic stock , averaging four composite indicators that measure consumption, econo social and environmental inequality , economic poverty, and finally economic insecurity. (The economic risks associated with unemployment, diseases, aging, etc., attention has been given to economic and social issues more than environmental cerns, as they are expressed by only two criteria, which are the costs of con

[1] -

environmental degradation as well as the per capita share of natural resources. The following table represents the most important components of the guide. Economic .imary formbeing, its pr-well

1 : BEAT BURGENMEIER, Political Economic Development Durable, published in Belgium, from Boeck , 2008, page 90.

being index according to the initial formulation-The economic well :(2-Figure (2

Standards	the components
.mption of goods/capitaConsu .Government/individual spending Metric unpaid labor (monetary .(value	**Consumption** (0.4)
Stock of Wealth The stock of an individual's physical capital .(money value) .(R&D per capita (monetary value capita Natural resources per .(monetary value) Human capital costs of education .(per capita :Subtract from the total the value of .Net external debt/capita Costs of environmental degradation the estimated social cost of) .(emitting carbon dioxide	Storehouse of wealth (0.1)
.vertySpread of po .Gini coefficient of inequality	**equality** (0.25)
.The unemployment Economic risks related to diseases .income families-Poverty of single .Poverty of elderly people	**Economic security** (0.25)

:Source Jean Gabrey , The cruissance au development, A la recherche d'indicateurs alternatives, futuribles no. 281- December 2002. www.futuribles.com being index -Both the modified gross domestic product index and the economic well reflect new experiences in the field of measuring growth and economic development, nded the horizon of measurement to aspects not included in traditional which expa indicators. They were also presented in the form of aggregate indicators or evidence in order to cover a larger number. Growth and economic development, however, did physical framework of measurement that was adopted in not depart from the traditional indicators, thus revealing a group of modern indicators that attempted to being and -include social and environmental aspects in Measuring economic well an Development and Compatibility Index calculating wealth, including the Hum Social and a group of environmental indicators, some of which we will learn about .below

The second requirement: indicators for measuring human development and sustainable development, a new vision for measurement

line with the intellectual development that has occurred in development models, a In set of indicators and indicators have emerged that reflect new interests, the most important of which are the Human Development Index and environmental aggregate .indicators

:Section One: Indicators for measuring human development

After we learned about the most important indicators that were used to measure growth and economic development and the most important alternatives that modified discuss the most important social (human) the objectives of the latter, we will measures and indicators that were found in order to measure human development, social progress and social harmony, according to Dr. Hamid Ammar . Social (human) uman condition in a society. In indicators are statistical tools to approximate the h their general function, they are symbols of the human standard of living, the quality of life, the process of change and social change, and they are the results of the impact .of modernization, growth and development efforts

Many attempts and works have emerged that sought, in their entirety, to know the extent to which economic development efforts reflect on the individual and on his being, many of which were included that were -most important components of well erlooked, such as health, education, housing, freedoms, gender equality, previously ov being -and other elements that no one can It denies its significant impact on the well of the individual. First: The Human Development Index. The initiative that the e up with as a result of the “Amerti Sun” research in 1990, in the United Nations cam context of searching for alternative indicators to income and distribution indicators, was considered one of the works that received much appreciation, as many believe pment Index is a model worthy of being considered. As an that the Human Develo alternative to economic indicators Traditionally, the Human Development Index :represents the average value of three basic elements

1. ;Longevity: measured by life expectancy. At birth
2. :ed by two variables of educational creditKnowledge: measur

أ. .Literacy rate among adults

ب. .Enrollment rate in primary, secondary and higher education

3. .Standard of living: measured by real GDP per capita

:equations Each of the three elements is calculated according to the following

1- minimum value) (minimum -Hope in life at birth (observed value . (maximum observed

2- Indicator of the level of education 3/2 (Indicator of schooling among adults) Indicator of schooling (Indicator of schooling in adults) Observed value 1/3 Study Guide (Observed value -mum Minimum Maximum value Mini - .(Minimum (Min. Maximum. Observed Value -Observed Value

3- -Lu(100)] / [Lu(maximum value)-Crude GDP index [Lu(N.D.H./capita) [(Lu(value) The world

Education level index 3/1 1/3 Human Development Index 3/1 Hope in life at birth . Output index Raw interior

indices was equal, which -We note that the distribution of weights among the sub means that each variable has the same impact on the final value of the index, and this .e of the same degree of importanceindicates that all variables ar

related development index-Second: Gender

Related Development Index (also known as the Gender Empowerment -The Gender Scale) uses constructed variables to measure the relative empowerment of men and and economic fields. Three variables are used to express this women in the political empowerment, two of them to express the economic aspect and the third to express :[1] The political aspect, and the three criteria are

1. managerial Percentage share of both women and men in administrative and ;positions
2. ;Percentage of each Of professional and artistic works
3. .Percentage of women's share. Of women and men in parliamentary seats

Third: Evidence of human poverty

s The concept of human poverty appeared in 1997. This concept expresses what i imposed from the outside and the most basic choices for human development, such as the opportunity to live a long, healthy and constructive life and enjoy a decent [2].respect and respect for others-standard of living, as well as freedom, dignity, self

21

.Tamimi, previous reference, p. 80-Raad Sami Abdul Razzaq Al 1-

Youssef Qureshi, Elias Ben Sassi, “Human Development Indicators, Concept, :2 Basics, Calculation,” International Forum on Human Development and Opportunities

[1] -
[2] -

dge Economy and Human Competencies, March for Integration into the Knowle .Ouargla University, Algeria, p. 41 ,2004

eht serusaem dna noitubirtsid eht fo erutcip a sevig xedni ytrevop namuh ehT“ accumulation of aspects of deprivation that still exist, This guide measures rms of the same basic human development dimensions as the deprivation in te development index Humanity. This index is a multidimensional measure of poverty. The ability to live longer -The Human Poverty Index is concerned with four axes: ;and in good health

- ;education
- ;omic resourcesecon
- .Share

The last two elements may be considered an addition made compared to the Human one for poor -Development Index. There are two indicators of human poverty differ in countries and the other for rich countries. They are similar in principle but terms of measurement standards. The following table shows us the differences and similarities between the Human Poverty Index for poor countries. and Economic .Cooperation Organization countries

for poor countries and Components of the human poverty index :(3-Table (2 developed countries

the poor countries	OECD countries	**Components of the Human Poverty Index**
The proportion of the population likely to die before the age of .years 60	Percentage of the population likely to die before the age of .years 40	Deprivation of living .long and healthy
Percentage of the population who cannot .write and read	Illiteracy rate among .the adult population	Deprivation Of .education
.Income threshold	Percentage of individuals with access .to drinking water Proportion of the population that does not have access to .basic health services Percentage of children under 5 years of age who suffer from .severe weight loss	.Economic average

The percentage of who are individuals able to work and have been unemployed for a period of less than 12 .Months		Participation or exclusion from life .Social

: Source**Mahi TABET-AOUL, op.Cit. p. 126-127.**

.Tamimi, previous reference, p. 81-Raad Sami Abdul Razzaq Al

Section Two: Indicators of social harmony

indicators express a new trend that differs from indicators measuring Social harmony human development. So, it is looking into new ways to measure the extent of agreement and harmony in the social fabric, and this is with interest By dividing measuring their social status by looking at... With society into age groups and .different categories for a set of variables

) First: Social Health IndexISS) .

It was prepared by a group of researchers at the American InstituteInstitute For Innovation In Fordham Social Policy -Work and research on it began since the mid " fifties of the last century. In 1996, researchers MarcMarque andLuisa Miringoff presented a social health index for countries as an alternative to gross domestic " n 1999 under the titleproduct. A book by researchers was published iThe Social Health Of The Nation siht fo noitaluclac eht dna ,dleif siht ni krow rieht snialpxe " guide is based on tracking the behavior of 16 secondary indicators over a certain in their entirety revolve around 5 period of time, which is rather long. The indicators basic issues that affect or are related to the most important age segments that make up society, so that they are given The value "0" represents poor performance, while the e. The following table summarizes maximum value "100" indicates better performanc .indices forming the Social Health Index-the most important axes and sub

Table (42): Components of the Social Health Index

the total	Elderly people	Adults	Young	children
Violence Fatal traffic accidents due to alcohol Accessibility On decent housing Inequality in the	Poverty at Segments over 65 years old Hope in life after 65 years	The unemployment Average monthly wage Percentage of coverage by health insurance and .social security	Suicide among young .people .Take drugs Leaving university .studies Single .mothers	Child .mortality Ill Treatment and care of .renchild Poverty among .children

distribution of income				

. : **Source**Jean Gabrey , op.cit

[1]**Second: Evidence of social insecurity**

specializing in applied studies of sustainable In December 2003, an institute development issues prepared a guide known as the Social Insecurity Guide. This guide consists of a set of indicators that measure and study the four most important have formed points of discussion over economic level, which-problems at the socio :the recent years that preceded the study. These four themes are

1. ;Labor market
2. ;Unequal distribution of income
3. ;debt impact
4. .Difficulties related to widely consumed materials

ranging from 0 to 10, and the index value indicators takes values-Each of the ten sub expresses the average of the ten partial indicators' values. The partial indicators of :the social insecurity index are distributed according to these four axes as follows

social insecurity index Indicators that make up the :(5-Table (2

Fourth axis: basic consumption	The third axis: debts	The second axis: income	axis : the labor market
Ratio of health 8 expenses to the individual's disposable income The evolution .9 of water and energy costs compared to the evolution of available per capita incomes Percentage .10 of private investments Administrative functions	The .6 development of the monthly cost ratio of mortgage debts. To the individual's monthly income The impact of .7 individual debts vidual on an indi .(Families)	The ratio of .3 the guaranteed minimum wage to the average .net wage Grant .4 Unemployment rate per capita. to average per .capita income Differences .5 between levels taxable -of non income	Number of .1 :unemployed The total number of unemployed and unemployed elderly people. The number of unemployed people who ve an recei unemployment .grant Wage .2 flexibility: the proportion of workers in

[1] -

ment manage) Public, education and health) to Gross .output			time -limited .jobs Percentage of workers in positions Temporary .work Number of furloughed .workers

Source: 07/28/2010http://www.ulb.ac/students/desge/cours/envif443:COUR08-4.indic

(adapted)

: http://www.ulb.ac/students/desge/cours/envif443:COUR08-4.indic 07/28/2010

the Social Insecurity Index have attempted to assess Both the Social Health Index and the problems The basic social indicators that create disparities in society and comparing these values with changes in the gross domestic product. However, these f criticism, especially the social health two guides have been subjected to a lot o index. The most prominent of these criticisms is the difficulty of accurately tracking changes in partial indicators, in addition to the slight change in some of them. That is, mpact than the change in another indicator, even if have a greater i -partial indicators .it is by a large percentage

Finally, we point out that the classification of traditional and modern economic indicators, human development indicators, and social harmony indicators was only to the study process and understand the changes that occurred in the facilitate .measurement process

Section Three: Environmental indicators

Integrating the environmental dimension has posed a real challenge in measuring ple Researchers who tried to find being for many peo-rates of development and well consensus formulas for a group of indicators of an environmental nature, and within this group we will discuss the most important indicators and indicators of an .environmental nature

First: The evidenceof real progress

n 1985, an institute in California proposed a new guide within the group of I alternative indicators for gross domestic product. The idea of the guide in general aims to reformulate the concept of wealth. It relies on the traditional method of th household consumption in addition to the various elements that calculating bo being are -contribute to the formation of wealth. Real wealth and economic well subtracted from it and the estimated value of wasted wealth, especially natural renewable and social resources -ral reserves and nonwealth, the destruction of natu such as the social costs of unemployment, violence, road accidents and inequality.

According to this methodology, national accounting systems are reformulated by t contribute to Increase or decrease in critically evaluating all the elements tha national wealth. The most important components of evidence of real progress can be .summarized in the following table

Components of the real evaluation guide (2-Table (6

Critical evaluation	Component elements of real eapplication evidenc
Positive value ePositive valu Positive value Positive value Negative value Negative value Positive value Negative value Negative value The final value of the directory	Personal consumption Adjusted personal consumption Domestic work (family and .(volunteer Services provided using sustainable goods. Positive value .Sustainable consumer goods costs Social costs (crimes, traffic ,unemployment, divorce accidents, time wasted going to .(...school). the job .Net investments Environmental costs (water pollution, sound pollution) are a negative value Ozone layer .(...problems .Net external debt .Evidence of real progress

: Sourcehttp://www.ulb.ac/students/desge/courses/envif443:COUR08-4.indic

Second: Theadjusted net savings index

This indicator aims to determine the value of economic resources, especially .environmental ones, i.e. natural capital

This is based on Calculating the national economic stock. This indicator depends o the total value of the mainly on adding and subtracting some monetary values t :[1] economic stock. The calculation method can be summarized by the equation next

Real Stock (Adjusted Net Savings Gross Savings + Education Expenditures risks - Estimated value of degradation of energy sources, mineral and forest resources .(from CO_2 emissions (social marginal cost of addressing one ton of emissions

1 -

evaluate and measure the true wealth of the national -This monetary index aims to re economy by modifying net fixed capital formation. The positive value of this index expresses that the consumption rate of current generations modified

1 : Cruise capabilities and development, presented by Pierre Duharcourt , France 2007.

of the national product remains acceptable and therefore it maintains a portion of productive capital sufficient for generations in the distribution of wealth available in a country. Justice between For future generations, this indicator expresses and must take into account, when analyzing the total values, the partial values of the variables of this indicator because they may differ in their behavior. Therefore, the overall rd: analysis must take into account the partial analysis of each of the components. Thi (Impact (Ecological Footprint.nineties, both NGOs have been -Since the mid workingredefining progress. progress And WWF) world wide Fund for Nature (prepared this indicator, which is considered primarily an environmental indicator, as o measure the amount of consumption by countries and individuals of it attempts t natural resources used for final consumption. The Ecological Impact Index is concerned with studying renewable natural resources exclusively, as it examines the rces are capable of production and renewal in light of high extent to which these resou consumption rates. The ecological impact is calculated by the area of land or seas and oceans necessary to provide the natural resources that humans use or those capable of ions in particular. Toxic and dangerous ones that are absorbing gaseous emiss .necessary to recycle waste From Various human activities

The ecological loss or ecological debt of a country expresses the difference between f this country, expressed the ecological impact and the natural production capacity o .[1] in the estimated area capable of natural production

For example The agricultural impact is calculated by estimating the total area of land needed for agricultural crops. As for the marine impact, it is determined by the amount of area needed to cover water production, and the energy determining impact by an estimate that represents the area necessary to provide the necessary energy. These estimates use a global unit of measurement, the total hectare or the tion per hectareaverage natural produchectare bio productif moyen and according to , WWF3 the global ecological (natural) impact It has almost doubled since 1961 until , so that it went from 120 ,1999% [2]which means exceeding the Earth's capacity in , roductive capacities for natural resources from its renewable terms of exploiting its p resources, and depriving the Earth of 20% l The figures .of its renewable resources for the year 2003 indicate that the impact The average ecological footprint of the billion total hectares, which is equivalent to 2.2 hectares per Earth's population is 1.4 capita. On the one hand, the average natural productive capacity of the Earth for the same year has been estimated at 1.8 total hectares. The average ecological impact per also been estimated. 4.8 for a total hectare Production capacity capita in Europe has

1 -
2 -

:The development indicators are durable, scientifically advanced, and durable, op.Cit.

2: Philippe Le Clézio, Indicators of sustainable development and environmental protection, economic, social and environmental advice, Paris 2009, page 31. www.conseil-économique-et-social- fr

3: Ibid , page 32

: Pierre Duharcourt , cruissance potentielle and development, op.cit.

European Naturally, it is equivalent to 2.2 total hectares, which means that the individual consumes twice the resources that the Earth can naturally produce. These indicators warn the world of the dangerous situation that consumption rates have reached, which today calls for a reconsideration of the methods of exploiting wable natural resources, especially by developed countries. The following table rene summarizes the development of ecological impact values from the year 1961 until .2005

Evolution of the ecological impact from 1961 to 2005 Unit: Total hectare (7-Table (2

2005	1990	1975	1961	
17,50	**14,50**	**11,20**	**7,00**	Ecological impact (billion (total hectares
4,13	3,81	3,63	3,40	Farmland
1,69	1,48	1,39	1,21	Pastoral lands
1,52	1,60	1,27	1,09	Forests
0,56	0,45	0,37	0,25	Marine fishing areas
9,11	6,83	4,22	0.83	Carbon footprint
0,44	0,34	0,27	0,20	The lands are allocated for construction
13,40	13,40	13,10	13,00	Natural production capacity

.:SourcePhilippe le Clézio, op.cit

notice from The table shows that the world's ecological impact has doubled from it was transferred from 7.00 to 17.5 billion hectares. We also ,2005 until now In 1961 note that the carbon footprint increased significantly, as in 1961 it reached 0.83 billion total hectares, while in 2005 it reached 9.11 total hectares. The following table .natural production capacity in the world represents the

Table (82): Ecological impact values in the world

Spaces allocated for construction	Marine fishing areas	Forests	**Pastures**	Farmland	Effect Carbon	**Countries Regions**
0,07	**0,09**	**0,23**	**0,26**	**0,64**	**1,41**	**World**

						(2,70)
0,13	0,61	0,17	0,28	4,04	1.15	-High income countries (6,40)
0,08	0,18	0,09	0,22	1,00	0,62	-Middle income countries (2.20)
0,05	0,02	0,15	0,09	0,44	0,26	-Low income countries (1,00)
0,05	**0,03**	**0,24**	**0,25**	**0,54**	**0,26**	**Africa (1,40)**
0,08	**0,08**	**0,04**	**0,08**	**0,69**	**1,34**	**Middle East and Central (2,30) Asia**
0,06	**0,13**	**0,07**	**0,08**	**0,49**	**0,78**	Asia countries) bordering the Pacific Ocean **(1.60)**
0,07	0,07	0,12	0,15	0,56	1.13	China (2,10)
0,04	0,01	0,10	0,01	0,40	0,33	India (0,90)
0,08	**0,10**	**0,32**	**0,72**	**0,57**	**0,65**	**Latin America and the Caribbean (2,40)**
0,10	**0,11**	1,02	**0,32**	**1,42**	**6,21**	**America ((9,20**
0,10	0,10	1,02	0,30	1,38	6,51	United States (9,40)
0,17	**0,10**	**0,48**	**0,19**	**1,17**	**2,58**	**Europe**

						European) Union) (4,70)
0,21	0,04	0,36	0,09	1,21	2.31	Germany (4,20)
0,25	0,17	0,39	0,32	1,28	2,52	France (4,90)
0,07	**0,17**	**0,29**	**0,04**	**0.94**	**2,00**	**Europe (the rest of the countries outside the Union) (3.50)**

.:Source Philippe le Clezio , op.cit

From the table we note that the United States of America is the number one consumer natural resources, the ecological impact per capita was estimated at in the world For total hectares, followed by Europe (the European Union) with 4.70 total hectares 9.40 .per capita

Fourth: Environmental Performance Guide and Environmental Sustainability [1]Guide

He developed both the Environmental Performance Indexand the Environmental) Sustainability GuideEnvironment Sustainability Index by Yale University in Columbia. The Environmental Performance Index aims to evaluate the effectiveness nmental policies compared to national or international goals. of a country's enviro This guide is based on 16 indicators related to public policies for the following basic topics: air quality, water resources, natural resources, Renewable energies, environment. To make these indicators comparable, each biodiversity, health and the one is rounded to a relative value on a scale ranging from 0 to 100, and the obtained points are grouped within a weighting system to obtain a final result representing a point out of100.

Environmental Sustainability Index is considered a tool for measuring the While the environmental performance of countries in the long term It reflects and translates the past, present and environmental future. It also integrates values related to natural pollution and the degree of deterioration of environmental systems. It is ,resources also concerned with current environmental policies as well as the ability of society to control and stop the negative effects of pollution. This index is calculated based on 76 ables representing 21 intermediate indicators from Among them: air and water vari quality, biodiversity, threats to ecosystems, waste, Natural resource management, .environmental policy

[1] -

ty Both the Environmental Performance Guide and the Environmental Sustainabili Guide aim to clarify environmental policies in countries and the extent of integration of efforts in various relevant fields. However, the final result provided by these guides depends on the selection of secondary indicators and the method of .ing weights that will determine the final resultsdistribut

The third topic: Sustainable development indicators: the United Nations vision towards an integrated model

There are dozens of models of indicators for measuring sustainable development, and difficult to address them, we will try Of all of them, during this research we since it is study the sustainable development indicators prepared by the United Nations as a practical model, and we chose this not because we believe that it is the best or best :rather it was in response to a group of reasons, which are model, but

1. .It is difficult to study other models because their number is very large
2. Considering the United Nations model as a reference for countries that enables ts priorities As well as its strategies them to select a number of indicators. , i ;within the framework of sustainable development
3. Considering the United Nations as an expert body in the field of preparing .sustainable development indicators

: www.insee.fr 03/15/2010

sustainable development indicators the emergence of **: requirement**

The United Nations is considered a pioneer in preparing sustainable development indicators. From the beginning, it has paid great attention to this subject. In 1995, a .drew up a special program sustainable development committee was appointed that

To prepare indicators Sustainable development has resulted in many attempts, the first of which was the result of cooperation between the Sustainable Development mic and Social Authority and the Statistics Authority of the United Nations Econo Affairs Authority, to which many organizations from within the United Nations later governmental -joined from and from various international governmental and non able bodies. This cooperation resulted in the preparation of the first set of sustain development indicators, numbering 134 indicators. Between 1996 and 1999, 22 countries implemented and experimented with them. This field experience confirmed the difficulty of the task of countries in adopting 134 indicators in an effective ich prompted the Commission on Sustainable Development to redouble manner, wh its efforts, as The number was later reduced until it reached 58 indicators distributed over a group of The basic topics and important subtopics addressed by sustainable itial version of the first group, which included 134 indicators, development. The in was published in what was known as the “Blue Book” in the first edition. This book or guide focused on explaining the new group of indicators. The United Nations elopment has identified a set of key issues upon Commission on Sustainable Dev which selection of sustainable development indicators is based, and these issues are .represented in the following table

Table (92): The main issues adopted in preparing sustainable development indicators .y the United Nationsb

Economic issues	Environmental issues	Social issues
.Economic structure Patterns of production . consumption and	. Atmosphere .Lands .Seas .Fresh water .Biodiversity	.Social equality .Public Health .education Living .Security .Population .

Abdel Rahim Muhammad Abdel Rahim, "Human Development and the **:Source** Ingredients for Achieving Sustainable Development in the Arab World," The Sixth Arab Conference on Environmental Management entitled Human Development and tainable Development, Egypt, Publications of the Arab Organization its Impact on Sus for Administrative Development, May**2007**.p. 11

The United Nations also presented a set of periodic reports and studies on indicators approaches in order to help countries Sustainable development is a set of practical develop and prepare their sets of national indicators for sustainable development. In a guide was prepared and published for countries that presented the scientific ,2007 nt indicators and explained to methods used in preparing sustainable developme countries users in general the methods of selection. The preference between indicators that are compatible with each case and how to calculate these indicators .and apply them in the field

United Nations is a comparative study in which Among the studies carried out by the it cooperated with both theEUROSTAT office and the Organization for Cooperation for Economic Development, on sustainable development indicators in about 25 country ranges between 12 and countries, where the number of indicators for each indicators, and during this report The most used indicators by the countries under 187 study were identified, which appeared in at least 10 countries, and this is what the .following table shows

ors most used in the European Union countries, Number of indicat :(10-Table (2 Australia and Canada

Repetition	**Indicators**	**Rank**
22	Warm gases	**1**
19	Levels of education achieved	**2**
18	Share per capita gross product	**3**
18	Classification and disposal of waste	**4**
18	Biodiversity	**5**

17	Official support for development	**6**
16	Unemployment rate	**7**
15	(Life expectations (living in a healthy state	**8**
15	Uses of energy from renewable sources	**9**
14	Risk of poverty	**10**
14	air pollution	**11**
14	Intensive energy use	**12**
14	Water quality	**13**
13	debt Net	**14**
13	Allocations from spending on research and development	**15**
13	Organic Agriculture	**16**
13	Area of protected lands	**17**
12	Deaths due to chronic diseases	**18**
12	Energy consumption	**19**
12	Unemployment rate	**20**
11	ozone layer Emission of gases harmful to the	**21**
11	Stocks of fish in biologically protected areas	**22**
10	Use of fertilizers and pesticides	**23**
10	(Transportation expenses by means (land, sea and air	**24**
10	Number of passengers according to transportation methods	**25**
10	intensity Water use	**26**
10	Forest areas and their uses	**27**

35 **:Source**Measuring Sustainable Development, 2007, op.cit. P

From the previous table, we notice that the greenhouse gas index came in first place, of gross domestic product followed by the education index, and the per capita share comes in third place, with the same rank in waste classification and disposal and biodiversity. These indicators reflect the three aspects of sustainable development, odels, it aims to which confirms that countries are always striving Through its m integrate and balance the three dimensions of sustainable development, without .focusing on one aspect and neglecting the other aspects

Second requirement: List of United Nations sustainable development indicators

institutions at the United Nations were not only concerned Sustainable development with preparing indicators for measurement, but also presented them Many from Periodic reports and studies to provide a set of practical approaches with the aim of and preparing a set of national indicators for assisting countries In developing sustainable development, we will present below two sets of sustainable development indicators. The first is considered the initial version of the indicators published in for the United Nations and was presented in while the second is the last set ,1996 .2007

from During the previous group of indicators, we find that the United Nations Commission on Sustainable Development used the dimensional approach in preparing these indicators, in addition to categorizing them based on theDSR approach tors of driving forces, situation, or response. Relying on as they are indica , the dimensional approach reflects the interest in measuring each dimension separately without addressing the existing interactions. Between the aspects bilaterally or r after the experience of nearly twenty countries that completely, it has become clea these indicators are not practical enough, which prompted specialists to work to reclassify and classify these indicators and add a new group in order to cover the up that was presented in 2007 by The United shortcomings. We will present the gro Nations Committee, through which development indicators were divided into 14 basic axes, are divided as shown in the following table. Table (122) The most ent indicatorsimportant axes of the United Nations sustainable developm

economical development	**natural disasters**	**the health**	**Poverty**
Economic relations International	Atmosphere	the earth	Governance
Consumption models And production	Biodiversity	Oceans, seas and beaches	education
		Fresh water	Population

. : SourceIndicators of Sustainable Development: Guidelines And Methodologies , 2007, Op.Cit

All models presented by the United Nations Commission on Sustainable Development are based on the most important axes and goals, as well as topics highlighted in Agenda 21, but the presentation is what differs from one group to aring Table No. (122) and Table No. (22 (13), we find another. To another, by comp that the presentation of the group The second differed from the first, whether in terms of number or in terms of the methodology used, as the committee in the last cators according to the dimensional approach, experiment abandoned dividing the indi as well as according to theDPSTR model In order for this group to be more .

effective for countries, the committee gave a set of methodological recommendations d, a set of procedures that must be to assist in the process. Application. To this en .followed in this regard have been described

The previous illustrative chart helps countries choose between the overall indicators presented, according to two dimensions: the availability of information and the ce of the indicator. The black boxes contain indicators that can be used by importan countries without any changes, while the dark gray boxes express indicators that can be redesigned according to the situation of each country. A country, either because ther indicators at the national level that are more important and more there are o interconnected than the proposed indicators or because there is not enough information and data available to calculate these indicators. As for the light gray tors that are important for countries but are not included in boxes, they include indica .the set provided by the United Nations Committee. For sustainable development

summary

The traditional indicators used to measure economic growth and development have terms of methodology and design, a new set of indicators undergone many changes In emerged to measure sustainable development, which is considered a new problem with large branches and extensions that traditional indicators could not cover and epare these indicators took approaches that measure. Therefore, each project to pr differ in their form and geometry, and through the previous study we find that it is usually Sustainable development indicators are presented according to three basic :forms

- ;often unrelated to each other An initial list of indicators that are
- A set of indicators classified according to the dimensions of sustainable ;development
- group from Composite and aggregate indicators that provide us with a component comprehensive measure of. Total phenomenon and phenomena Its .part

Among the most widely used methods in engineering sustainable development :indicators we find

- ,Engineering according to the main subjects or fields
- ;Engineering according to the dimensions of sustainable development
- ;types of capital Engineering according to
- .Engineering according to strategic objectives

And from Through the set of indicators for measuring sustainable development that :we presented, we conclude that they were presented within Two main perspectives

- are those indicators that are calculated by Aggregate indicators: These collecting a set of weighted secondary indicators that reflect the basic variables of the dimensions of the phenomenon. Among the negative points that are ting the secondary taken into account in this method is the method of selec indicators, as well as the method of distributing weights among them and the methods of unifying the values of these indicators. Within a common ladder. Among the guides that fall within this group are the Environmental .and the Environmental Sustainability Guide Performance Guide
- Overall indicators: These are those that start from existing traditional indicators and attempt to modify them by adding or subtracting a new set of elements that enomenon studied. Perhaps are consistent with the new dimensions of the ph being -the most prominent of these attempts are the evidence of economic well .and the evidence of real savings

It remains for each country to choose the method and methodology that is compatible the case study, we will try to learn about the case of with its specificities, and through Algeria and the approaches it chose in order to prepare measurement indicators. .sustainable development

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Chapter five

Chaper five : input and output analysis

Before going into the details of input and output analysis, it should be noted that the main features that distinguish it from other quantitative analytical tools are mentioned, input and output analysis is the best alternative to general equilibrium.

1-2 - Demand is an external variable: In the analysis of inputs and outputs, demand is considered a variable outside the model. It deals with the technical problems of production;

2.2 Absorption capacity the capacity of input and output analysis to accommodate areas outside the field of economics such as pollution and the environment;

Exclusively with

3.2 Aggregation capacity the versatility of input and output analysis in their applications at different levels of aggregation as an economic sector, such as a region as a country or even at the international level (global input and output analysis);

4.2 Macro level: Input-output technology allows relationships within the economic system to be analyzed as a whole rather than individual elements, not applicable to partial equilibrium analysis).

5.2 General equilibrium Input-output analysis focuses on an economy that is in equilibrium. It relies on empirical investigation, rather than mathematical formulations and their complexities.

Third: Input-output models

Economic theory is an abstraction of the complex reality due to the overlapping and intertwined relationships, difficult to understand once, and trying to formulate economic life in all its complexities and subtleties in a mathematical model is intractable, so abstraction and simple hypotheses are necessary. These interrelationships are not as important, so the economist takes a picture of the real economic phenomenon and conducts a study on it and draws conclusions This image is known as the economic model.

The economic model is just an approximate theoretical structure of the actual economy, it is a relationship for a set of mathematical symbols that reflect the actual economic interdependence, input and output models provide the researcher with tools to test the synaptic relationships within the country's economy between sectors, sectors and end consumers. One of the objectives of input-output models is to monitor and track all cross-sector exchanges in order to track how changes in one or more sectors affect the rest of the national economy. The input and output analysis consists of two parts:

Chaper five : input and output analysis

Descriptive aspect: Preparing and drafting the inputs and outputs table, which is considered an accounting framework in accordance with the accounting system in force in that country.

Analytical aspect: which makes the table of inputs and outputs that accounting system of value and convert it from a descriptive table to a mathematical model of inputs and outputs, then an analytical tool and a strong predictive model is a set of calculations that are necessary, and hypotheses that must be formulated first.

1- Types of input and output models

In input and output models, the economic structure is described and defined in terms of industries (or sectors producing goods and services for intermediate consumption (meeting the needs of productive sectors to carry out the production process) and (meeting the needs of the basic components of final goods and services) final demand.

1-1- Final application variable (inside or outside the form)

In the analysis of inputs and outputs into two different models according to the final application component, it can be distinguished as an element within the model, i.e. determined from within the model. Or as an external element which specifies outside the form. And on it there is a closed model and an open model. Final demand (consumption) usually consists of key elements such as households and non-profit enterprises, public spending and investments (which include crude fixed capital formation and inventory changes) and exports.

Thus, in Leontief models it can be distinguished as a description of an economy where inputs are equal to outputs in the sense of
(consumption = production).

Leontief models are of two types:

A. closed model where all production is consumed by the participants in production.
B. An open model where part is consumed internally and the rest by external sectors.

A- Closed Form:

The intersectoral structure is considered internal, i.e. its behavior is determined by the final demand within the system (model). The elements of the final demand (or its components are usually the expenditure component of families) as a sector like the rest of the economic sectors, and is integrated into the cross-sectoral matrix through new row and column rays, to describe how interdependence is made by different sectors, relative income is sought For each contributor to the system, considering that all inputs and outputs are used within the system, for example, the families sector is an element) (work within as a new sector (industry) (families produce work and are determined within the model and not outside it). Therefore, the sum of each column in the direct expenditure matrix A is equal to 1. There is no surplus (0 = y) and the technical (technological) equation of the closed Leontief model is as follows:

Chaper five : input and output analysis

(I-A) X = 0

where 0 is the zero-perpendicular ray (vector) This equation of the matrix represents a system of equations whose right-hand side is always non-existent called the system of homogeneous equations in the closed model The technical equation has no single solution and therefore has a single marginal solution.

$X_1=X_2=. . . =Xn=0$

In the closed model, the technical equation has no single solution and therefore I – A) The direct and indirect expense matrix does not exist and cannot be used to find the solution, nor by the Cramer rule method in matrix algebra, in this case the solution can be single, but marginal under a homogeneous system with a non-singular matrix A 24. As mentioned in the first equation of Chapter 1 (1-1) that the value of uses (the sum of the column) is equivalent to the value of sales (the sum of the line), and that the sum of operating costs equals the production elements with the primary production elements (added value), which represents the expenditure of the product. Accordingly, the price of each product can be built from the prices of the intermediate inputs used for its production, and therefore to produce one unit of the first product (1) inputs must be provided (a11 + ... + an1 + 21 + then if P1 is the price per unit of the first product (1), then the cost of intermediate inputs for the production of 1 unit of the first product (1) is mathematically written as follows:

$_{P1}A_{11}+P_2A_{21}+.....P_1A_{I1}+P_NA_{N1}$

where the right side represents the price paid by the first sector for the used goods and the left side P1 represents the income of the first sector. So the difference between the cost of one unit of intermediate inputs for the first product and the price per unit is the added value of one unit of the first product. Hence, the mathematical formula is as follows:

(P1-P1 a11+ P2 a21+. .. + P¡ a¡1 +. .. + Pnan1) = V1

where the left side represents the price paid by the first sector for second-hand goods and the left side 71 represents the income of the first sector. The general formula for a system with n products is as follows:

$$p_1 - (p_1 a_{11} + p_2 a_{21} + . \; .. \; + p_i a_{i1} + . \; .. \; + p_n a_{n1}) = v_1$$

$$p_2 - (p_1 a_{12} + p_2 a_{22} + . \; .. \; + p_i a_{i2} + . \; .. \; + p_n a_{n1}) = v_2$$

Make some rearrangement to facilitate the process of resolving the price system, the following:

$$p_n - (p_1 a_{11} + p_2 a_{21} + . \; .. \; + p_i a_{in} + . \; .. \; + p_n a_{n1}) = v_n$$

$$(1 - a_{11})\, p_1 - p_2 a_{21} - . \; . \; . - p_i a_{i1-} . \; . \; . - p_n a_{n1} = v_1$$

$$- p_1 a_{12} + (1 - a_{22})\, p_2 - . \; . \; . - p_i a_{i2} - . \; .. \; - p_n a_{n1} = v_2$$

$$\ldots \qquad \ldots \qquad \ldots$$

$$- p_1 a_{11} - p_2 a_{21} - .. \; . - p_i a_{in} - .... + (1 - a_{n1})\, p_n = v_n$$

If this system is written in the form of matrices, it is:

$$\begin{bmatrix} (1-a_{11}) & -a_{21} & -a_{i1} & -a_{n1} \\ -a_{12} & (1-a_{22}) & -a_{i2} & -a_{n2} \\ \dots & \dots & \dots & \\ -a_{1n} & -a_{2n} & -a_{in} & (1-a_{nn}) \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \dots \\ p_n \end{bmatrix} \begin{matrix} = \\ = \\ \\ = \end{matrix} \begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{bmatrix}$$

A closer look at the matrix (al-1) finds that it is only a transposed (switched) matrix of coefficients A Writing in the form of matrices in general is:
(I-A) P=V

and from it to find P by the method of inverse matrix:
$P=(I-A^t)^{-t}v$

One of the properties of determinants and the inverse of the matrix is that the inverse of matrix A is the same as the inverse of its switch:
$A\text{-}1=(A^t)^{-1}$

And on it the inverse of Leontiv
$(I-A)^{-1}=(A^t)^{-1}$
And the price vector is calculated by the Leonitive inverse
Corresponding to the vector of the result
$P=(A^t)^{-1}V$
$X=(A^t)^{-1}y$
where P represents the price line beam and X represents the output column vector for each output in the system The equation above (2-6) represents the needs where income equals spending. Price equation (92) allows calculating the effects on the price level of a change in value added by one unit like equation (2-12).

B- Open Model In the open model: the economic system is divided into two main sectors. The intermediate demand sector includes all productive sectors, and the final demand sector with all its components, which is determined outside the model. That is, in addition to the intermediary internal consumption of goods and services by the productive sectors, there is external demand for them as well, the remaining part of the goods and services is called final demand or surplus goes to external consumption (family sector government sector export). The mathematical treatment is as follows: if this excess is placed in the perpendicular ray (vector) of, the technical equation of the open input and output model.
x - Ax = y
Or
I (I– A) x= y
The inverse (I) - (A) can always be used to solve the technical equation provided that all the inputs of A (aij) are positive and the sum of each column is less than 1. The solution has economic meaning only if the components of X are non-negative. In this

case, the sum of the elements of the column of the direct (technical) coefficient matrix (A) becomes) is less than one.

$(1>\sum_{i=1}^{n} a_{ij})$

$$X = (I - A)^{-1} Y$$

The sum of the column under the open model represents the partial cost of inputs without the main inputs paid for one unit produced in Sector L.

$$\sum_{i=1}^{n} a_{ij} < 1 \; (j = 1،2، \ldots n)$$

Therefore, the value of the main inputs paid in the sector for one unit of its production is given in the form
following:

$$1 - \sum_{i=1}^{n} a_{ij}$$

The equilibrium equation for the first sector is as follows:

$$X_1 = a_{11} x_1 - a_{12} x_2 \ldots - a_{1j} x_j \ldots - a_{1n} x_n + y_1$$

أو

$$(1 - a_{11}) x_1 - a_{12} x_2 \ldots - a_{1j} x_j \ldots - a_{1n} x_n = y_1$$

The same procedure is followed for equations similar to the rest of the sectors, so the following set of equations is followed:

$$- a_{21} x_1 + (1 - a_{22}) x_2 \ldots - a_{2j} x_j \ldots - a_{2n} x_n = y_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$- a_{i1} x_1 - a_{i2} x_2 \ldots + (1 - a_{ij}) x_j \ldots - a_{in} x_n = y_i$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$- a_{n1} x_1 - a_{n2} x_2 \ldots - a_{nj} x_j \ldots + (1 - a_{nn}) x_n = y_n$$

The matrix formula, the set of equations for the system as a whole is as follows:

$$\begin{bmatrix} (1-a_{11}) & -a_{12} \ldots & -a_{1j} \ldots & -a_{1n} \\ -a_{21} & (1-a_{22}) \ldots & -a_{2j} \ldots & -a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ -a_{i1} & -a_{i2} \ldots & (1-a_{ij}) \ldots & -a_{in} \\ \ldots & \ldots & \ldots & \ldots \\ -a_{n1} & -a_{n2} \ldots & -a_{nj} \ldots & (1-a_{nn}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \\ xi \\ \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \\ y_i \\ \\ y_n \end{bmatrix}$$

Looking closely at the left side of the equations, the following can be observed:
If the number 1 " in the diameter of the matrix is overlooked, it is in front of the exchange matrix "A", but it
negative (A-);
the unit matrix with a diameter of "1" and the top of the diameter and the bottom of which are zeros "0";
As if this right-hand side is the unit matrix minus the exchange matrix (I-A).
Therefore

It can be written in the form of an abbreviated matrix as above:

)I - A)X=Y

The inverse (I -A, if any), can be used to solve the technical equation, provided that . All the places of A, namely (aij), are positive, the sum of each column is less than "1", and the matrix A is not singular. As long as (I - A) is not singular and squared, the inverse can be found I - A and obtain a single solution to the previous system of linear equations from the reality of the known base equation:

(I-A)X=Y

X = AX + Y X-AX=Y (I-A)´1(I-A) X= (I-A)-1Y

IX=(I-A)-Y

At last

X = (I - A)Y

The solution has economic meaning only if the components of X are non-negative Equation (12-2) is the backbone of the input-output model where the matrix (I-A) is called the final order coefficients matrix or the multiples matrix. Given this matrix [1-] (I-A), the production values X are functions of the final demand Y on various sectors. It can be formulated mathematically as follows:

X = F(y)

Conclusion Use one of the two types of open form where the final demand sector is determined from outside the system, or a closed form where the final demand is determined from within (the system depends on the planned objectives and intent of the economic analysis.

2.1 Static and dynamic input and output models by time

If we take time into account, the input and output models can be viewed from this angle, and we have static and dynamic ones. Kinetic and static model Taking into account the time factor, when building the model, the change in investments is studied and followed up chronologically, and structural relationships are studied in view of the change in inventory as well as the formation of fixed capital, so we are facing a moving model or not taking the change over time into account in the analysis of investments, the model is calculated for a specific period of time one year, the model analyzes the relationships The structure with interdependence between the needs of the sectors and their outputs during a specific period of time, so we are facing a static (fixed) model. So, there are two static models and a kinetic model.

A- The closed Static model

Leontief began his first work with a static and closed model, static in the sense of excluding the time factor (the effect of capital formation). It is closed that all variables are determined within the model, i.e. all production is consumed by the participants in production. The relative income of each shareholder in the system is searched for as all inputs and outputs are used within the system, so that, for example, the family’s sector (work) enters as a new sector (industry) (families produce work), and this is what was explained above when addressing the closed model in all details.

B - Open static model: The open Static model

Besides sectors (n) If an open sector is added that determines an external final demand (e.g. the family's sector), it represents a key input (labor) The model becomes open, without taking into account change over time in the analysis of investments, the model is calculated for a specific period of time. The model analyzes the structural interdependent relationships between the needs and outputs of sectors over a specified period of time, forming a static (static) model. So, in our open static model the final demand with all its components determines the output Prototype. On the one hand, the analysis of the structure of the economy will be for a specific period. This is what was explained above when going into the open form in all the details.

C- The dynamic input-output model)

The most important characteristic of the dynamic model (kinetic) that it takes into account the element of time, and it also takes into account the changes in the economic variables that make up the model, the realistic view of the process of production of fixed capital and its uses in the productive field reveals the existence of a time lag between the production of capital on the one hand and its uses in economic activities on the other hand (25). The relationships arising between sectors of the exchange of goods and services reflect the level of technology in a particular country or region. Technology is an essential feature of Input-output analysis The investigation seeks to determine what can be produced and the quantity of each intermediate product to be used in the production process. Given the number of resources available and the level of technology, growth in one sector can be due to (or provoked) growth in other sectors. The input and output methods allow the deconstruction of the impact of this interdependence and the growth components are constantly defined.

One of the concerns of the input-output economy is that they are specific models for the use of empirical data and that they are coherent. All internal changes to the input-output table are the result of changes in external sectors. In the static input-output model, the economic analyst deals with production only or the "current account" aspect of the economy (operational processes rather than investment operations) which gives an accurate example of model coherence. Investments and capital activities are not listed and are generally included in the final demand components mentioned earlier, not in the Input-output Exchange-Matrix section of intersectoral flows. Perhaps this becomes a serious constraint on static input-output models, due to changes in the capital structure of the sector and changes in the pattern of sources of capital equipment that are among the most important manifestations of technological change that have direct effects on output growth.

Economies are so dynamic that kinetic input-output models must be used.

Previous models of inputs and outputs were concerned with how to model flows of goods and services in the economy, and how intermediate demand for goods and services affects the overall demand for goods and services as well as the use of unproductive inputs such as labor. But in other models of production theory, the term "capital" is used as a variable, which helps in understanding the level of output in an

economy that can be reached and achieved, and this is especially important in the theory of economic growth and economic development, so the variable "capital" can be introduced in the input-output models as follows:
The important thing that needs to be the term "capital" is clear and what it means; in previous models that have passed with us, capital has always been seen as a different element from output, output is an inflow while capital is considered a stock and is implicitly processed into the elements of final demand, but this is not a way to deal with things, especially the term capital is frequently used if development is taken into account in the long run. But it is not clear that capital can be considered as a "stock", and the reason that capital wears out and disappears due to technological progress and then replaces capital is an influx but is flowing relatively slowly. Distinguish between these commodities that make up flows and those goods that make up inventories depend mainly on the time factor used Some slow-moving goods look like stock in the short term, but like flows in the long term. In the very long run all things seem to flow. This problem is widespread and is known in the accounting profession which enters the stock of outputs and inputs as capital means productive factors. In light of the above, it is clear that it is not necessary to specifically introduce the "capital goods" sector into the economy. On the other hand, all goods can be treated as having some "inventory" characteristics and features.
- Scalar example:
For the purpose of illustrating how to use the input-output model, assuming an economy consisting of three sectors shown in the following table: Table 05:

جدول رقم 05: **اقتصاد مفترض** الوحدة مليون دج

القطاع	1	2	3	Y	X
1	80	30	45	115	270
2	35	40	35	125	235
3	37	18	32	93	180

The matrix of direct technical coefficients is as follows, noting that the sum of each column is less than one.

$$A = \begin{bmatrix} 0.2963 & 0.1276 & 0.25 \\ 0.1296 & 0.1702 & 0.1944 \\ 0.137 & 0.0766 & 0.1778 \end{bmatrix}$$

$\sum_{i=1}^{3} a_{i1} < 1$ $\sum_{i=1}^{3} a_{i2} < 1$ $\sum_{i=1}^{3} a_{i3} < 1$

If the main inputs are denoted by the symbol (ao), and let it be in the example, this is what pays families for the work done to produce one unit of the sector's commodity. and therefore, the value of these primary inputs is:

$a_{01} = 0.4371$ $a_{02} = 0.6256$ $a_{03} = 0.3778$

$$\begin{bmatrix} 0.7037 & -0.1276 & -0.25 \\ 0.1296- & 0.8298 & -0.1944 \\ -0.137 & -0.0766 & 0.8222 \end{bmatrix}$$

Therefore, the open input and output model A(X=Y- I can be expressed)

By retaining the parametric image of the final request ray Y, the solution is in a formula that enables different solutions to be obtained.
Calculate the different values given to the final request beam each time.
For example, if you give values forY, which are in the table above for the numerical example, namely:
y2 = 145 _y1 = 150 = y3 = 110
For example, let it be a target end product within a given development program and by finding the inverse of Untiev.

1.582455	0.2941749	0.550719
0.3158171	1.29070899	0.401202
0.29310135	0.1692657	1.345691

The system can be solved by the above equations:

$$\begin{bmatrix} 1.582455 & 0.2941749 & 0.550719 \\ 0.3158171 & 1.2907899 & 0.401202 \\ 0.29310135 & 0.1692657 & 1.345691 \end{bmatrix} \begin{bmatrix} 150 \\ 145 \\ 110 \end{bmatrix} = \begin{bmatrix} 340.602701 \\ 278.66932 \\ 216.534739 \end{bmatrix}$$

X_3 =216.534739، X_2= 278.66932 ، X_1= 340.602701 فتكون قيم الحل محددة

The following question comes: if the production of combination 1 and 2 was under specific quantities of primary inputs, will the necessary quantities of inputs be in line with. What is available in economics? Since the coefficient values for the initial inputs are the following:

$a_{01} = 0.4371$ $a_{02} = 0.6256$ $a_{03} = 0.3778$

The necessary quantity of the main input can be calculated as follows:

$$\sum_{j=1}^{3} a_{0j} X_j = 0.4371 * (340.602701) + 0.6256 *(278.66932) +0.3778$$

$$*(216.534739) = 148.877441+ 174.335527+81.8068244=405.019792$$

If the supposed economy can save at least 405 million dinars in value for primary inputs, the aforementioned final demand beam is possible.
In the reverse case that the quantity available is less, the target level of production must be reduced. This is an important feature of the power of the Input-Output Analysis Tool, as long as the direct technical coefficient matrix (A) is constant, even in the short term, the Leontief inverse matrix 1-(I-A)) also remain constant, hence the large number of rays can be examined and tested for the final application as different alternatives targeted to the decision-maker for several development programs. This shortens the time and effort to solve the model, where the work is limited to finding the inverse of Leontief and the new final request beam.

Chaper five : input and output analysis

Input-output model assumptions:
In order to transform the descriptive model into a predictive model, an important set of assumptions were made for simplicity. The economy is divided into n different sectors, each producing a single homogeneous output. Which means using a single technical method. Therefore, n sector n has a different product. As well as assuming the stability of technical transactions and input prices.
-1- Assumption of homogeneity Two products are not produced jointly. Each sector produces only one homogeneous product; Achieving homogeneity requires the following:
The possibility of complete substitution between the components of a single homogeneous commodity;
B- Exclude the possibility of substituting products in one sector with those of other sectors. Or, in other words, the exclusion of substitution between the products of the different productive sectors of the national economy;
c. The use of a single artistic method for the production of any of the goods produced in the sector or industry 26".

2- Assumption of the stability of technological transactions: Production functions in sectors are linearity The meaning of linearity is that the inputs are proportional to the output (production). Each sector uses its inputs in a fixed ratio for its production denoted by aj. Production functions are stable in the short term and linear. The inputs remain in a constant ratio with the output level. It means that there is no alternative between different materials and there is no technological progress. There are constant input coefficients for production. This assumption (2) includes yield stability relative to Constant returns to scale in the sense of every change in each input by K will lead to a change in the volume of production by the same percentage K, if the inputs are doubled, the production is doubled. This assumption is known as proportionality It is assumed that the relationship that is realized in practice between inputs (inputs) and production (outputs) is linear and homogeneous that does not allow substitution. Between the inputs assumption of constancy this means:
A- The principle of stability in the relationship: What is meant by stability, in the relationship. with the volume of production and not the stability over time;
B- Stability of ratios: The mixing ratios of production inputs should be fixed, and this requires the use of the lowest possible amount of all production inputs produced in the sector, assuming the stability of the mixing ratios of female users results in the following:

- Stability of volume yields where production increases in the same proportion as all production inputs;
- Convexity of the curves of the equivalent product (Isoquants) towards the origin point.

That is, the set or map of the curves of the equivalent product takes the right-angled Ashi shape. Nested Right-Angled Corners is an indication that if new units of one factor of production are added without changing the units of the factor or factors (the other other), there will be nothing new in the quantity produced, including the absence of substitution within the production process 27.

3- Assuming the relative stability of prices of production inputs: At least in the short term, the change in the prices of inputs causes a change in technical transactions.

Fourth: Mathematical framework for input and output analysis

1- The mathematical structure of the input and output system:

In the input-output model, the economy is divided into n interacting sectors. Each sector offers as a buyer of raw materials, semi-finished and finished materials from other sectors, work from families and if it is not enough local purchases
Imports from abroad. On the other hand, each sector sells its products to other sectors (intermediate products), final demand sectors (finished products), families, companies, government and foreign trade.

Sector A GDP = Intermediate Use + End Use

If we denote by Zij what sector J needs as inputs from the production sector one unit of production of sector J; by Yi to the final demand for the product of sector A; and by Xi to the total output of the sector. A. The system consists of a set of n A linear equation with n variable (unknown), so matrix algebra plays a big role in factoring and trying to solve a single system of simultaneous equations:

- If the economy is divided into n sectors (industry), taking into account the complete homogeneity of productive activities within the same sector. The necessary information is the flows of goods and services from the productive sectors to the sectors used, this sectoral entanglement of flows is measured for a certain period, usually a year in monetary terms.

Exchanges between sectors of procurement and sales are in kind (tons of iron, seeds) accounting can be recorded in kind (natural) or cash (value). Due to the problems that accompany accounting registration in kind, it is advisable to record at value despite problems of price change.

- If we denote by Xi the total output of the sector ..
- If we denote by Yi the final demand for the output of sector A .
- If we denote by Zij what Sector G needs as inputs from the production sector one unit of sector J production, then the equation that reflects sales from sector A production to all sectors n is as follows:

$$[X_1 = Z_{11} + Z_{12} + \dots Z_{1j} + \dots Z_{1n} + Y_1]$$

$$[X_2 = Z_{21} + Z_{22} + \dots Z_{2j} + \dots Z_{2n} + Y_2]$$

....

$$[X_i = Z_{i1} + Z_{i2} + \dots Z_{ij} + \dots Z_{in} + Y_i]$$

....

$$[X_n = Z_{n1} + Z_{n2} + \dots Z_{nj} + \dots Z_{nn} + Y_n]$$

Chaper five : input and output analysis

The first directory i] indicates the sector that produced this entry.
[The second directory [ ] indicates the sector that used this entry.
Based on the previous hypotheses, one unit of the commodity of sector J needs a fixed quantity (value) of the commodity of sector A as an input and we denote it with the symbol (aij) and refers to the input coefficient (user) Technical coefficient Thus, the technical (technological) transactions in an economy represent the character (structure) of that economy from a technical point of view, or the relationships between the production of goods of various types and the various factors needed by this production, and it is enough to know these Transactions Because we directly determine what uses are necessary to produce a certain quantity or value of goods or a quantity or value of a group of goods in a particular sector and therefore Sector A must produce:

$$a_{i1} x_1 + a_{i2} x_2 + \dots\dots + a_{ij} x_j + \dots\dots + a_{in} x_n$$

In order for this sector A to meet the needs of other sectors. Since the economy has n sectors, AIJ can be arranged in a matrix.
A] = aij] and refers to technical parameters or direct flow matrix
Matrices of input coefficients
Input

المخرجات	1	2	J	n
1	a_{11}	a_{12}	a_{1j}	a_{1n}
2	a_{21}	a_{22}	a_{2j}	a_{2n}
i	a_{i1}	a_{i2}	a_{ij}	a_{in}
		. . .		
n	a_{n1}	a_{n2}	a_{nj}	a_{nn}

Since the input and output coefficients are constant, then

zij = aij xj و عليه aij = zij / xj

Therefore, the equations become:
X1 = a11 X1 + a12 X2 + . . . + a1c Xc + ... + a1n Xn + y1
X2 = a21 X1 + a22 X2 + . + a2j Xj + . + a2n Xn + y2
Xi = ail X1 + a2 X2 + . + aij Xj + ... + ain Xn + yi
Xn 1 = an1 X1 + an2 X2 + . + ang Xj + . + ann Xn + Yi
These rates also serve to show the correlation of sector entanglement flows as well as enable the answer to the specific question "What level of production must each of the industries achieve in the economy, provided that there is the final demand for this production?" In other words, if demand for external sectors is predicted by specific amounts for the coming year, what production is needed from each sector to meet this final demand? Accordingly, with Information

Chaper five : input and output analysis

كمجاهيل يمكن إيجادها X_1 ، X_2 ،X_n . . . . . فإن a_{ij} وكذلك Y_1 ، Y_2 ،Y_n . . . . . .

$$X_i - \sum_{j=1}^{n} a_{ij} X_j = Y_i \qquad (27\text{-}2)$$

$x_1 - a_{11} x1 - a_{12} x_2 - \dots\ a_{1j} x_j - \dots\ a_{1n} x_n = y_1$

$x_2 - a_{21} x1 - a_{22} x_2 - \dots\ a2j\ xj - \dots - a2n\ xn = y2$

.

.

$x_i - a_{i1} x1 - ai2\ x2 - \dots - aij\ xj - \dots - ain\ xn = yi \qquad (28\text{-}2)$

.

.

$x_n - a_{n1} x1 - an2\ x2 - \dots - anj\ xj - \dots - ann\ xn = yn$

The system of equations is as follows: and makes x1 together for the first equation, xz together for the second equation, and xn together for the equation n

$(1 - a_{11}) x_1 - a12\ x2 - \dots - a_{1j} x_j - \dots - a_{1n} x_n = y_1$

$- a_{21} x_1 + (1 - a_{22}) x_2 - \dots - a_{2j} x_j - \dots - a_{2n} x_n = y_2$

. ...

$-a_{i1} x_1 - a_{i2} x_2 - \dots + (1 - a_{ij}) x_j - \dots a_{in} x_n = y_i$

....

$- a_{n1} x_1 - a_{n2} x_2 - \dots - a_{nj} x_j - \dots + (1 - a_{nn}) x_n = y_n$

The matrix formula is as follows:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nn} \end{pmatrix} ، \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \\ xi \\ \\ x_n \end{pmatrix} ، \quad y = \begin{pmatrix} y_1 \\ y_2 \\ \\ y_i \\ \\ y_n \end{pmatrix}$$

The perpendicular vector of the final order.
X: Vertical vector of domestic production.
A: Technical Parameter Matrix
In general, the last X = AX + Y where

$(2\text{–}30) X\text{–}AX = Y$

Production balance equation

They are called production scales. It is the basic equation in the analysis of inputs and outputs and represents the rows in the matrix

The production side and the columns in the matrix are the costs side. The equations associated with the above equations are:

$$p_j - \sum_{i=1}^{n} a_{ij}\, p_i = v_j \quad (31\text{-}2)$$

This equation determines the price values in the economic system under study.
PJ : Unit price of sector G products.

Vi : Value added per unit of products of Sector J .
aij Pi : input value of sectors (n) for the production of one unit of sector g.
Accordingly, in terms of quantities:
In the form of quantities

$$(32\text{-}2) \qquad \mathbf{xi} - \sum_{j=1}^{n} \boldsymbol{a_{ij}} \qquad \mathbf{Pi = yj}$$

In the form of values (prices)

$$(33\text{-}2) \qquad \mathbf{Pj} - \sum_{i=1}^{n} \boldsymbol{a_{ij}} \quad \mathbf{Pi = yj}$$

We calculate the quantities at prices and prices in quantities, so they are:

Calculating the substitution of equation (322) in Pi and taking the sum over a.
Calculating the substitution of equation (2-33) in X and taking the sum on j.

$$\sum_{i=1}^{n} p_i \; x_i - \sum_{n=1}^{n} \sum_{j=1}^{n} a_{ij} \; p_i \, x_j \; = \sum_{i=1}^{n} p_i \; y_i$$

$$\sum_{j=1}^{n} p_j \; x_j - \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \; p_i \, x_j \; = \sum_{j=1}^{n} v_j \, x_j$$

Accordingly:

$$(34\text{-}2) \qquad \mathbf{GDP} = \sum_{i=1}^{n} P_i \, y_i = \sum_{j=1}^{n} v_j x_{j\text{j}}$$

This GDP is known as

The sum of the final order is equal to the sum of the total added values in the matrix form:

في صورة كميات
$$\begin{cases} x - Ax = y \\ (I - A)\, x = y \end{cases} \qquad (35\text{-}2)$$

في صورة قيم
$$\begin{cases} P - PA = V \\ P\,(I - A) = V \end{cases} \qquad (36\text{-}2)$$

where | The unit matrix and assuming the inverse (I - A) exists:

$$(I - A)\, x\, (I - A)^{-1}\, y \qquad\qquad (I - A)^{-1}$$

$$I\, x = (I - A)^{-1}\, y$$

$$x = (I - A)^{-1}\, y$$

A segment with the formula of algebra of matrices for n

$$\begin{bmatrix} (1 - a_{11}) & - a_{12} \; . \; . \; . \; . - a_{1j} & - a_{1n} \\ - a_{21} & (1 - a_{22}) \; . \; . \; . \; . - a_{2j} & - a_{2n} \\ \\ - a_{i1} & - a_{i2} \; . \; . \; . \; . \; (1 - a_{ij}) & - a_{in} \\ . & . & \\ . & . & \\ . & . & \\ - a_{n1} & - a_{n2} \quad - a_{nj} & (1 - a_{nn}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \\ xi \\ \\ \\ \\ x_n \end{bmatrix} \begin{vmatrix} y_1 \\ y_2 \\ \\ yi \\ \\ \\ \\ y_n \end{vmatrix} \qquad (37\text{-}2)$$

$$(P_1, P_2 \; . \; . \; . \; P_n) \begin{bmatrix} 1 - a_{11} & - a_{12} & - a_{1n} \\ - a_{21} & 1 - a_{22} & - a_{2n} \\ \\ - a_{i1} & - a_{i2} & - a_{in} \\ - a_{n1} & - a_{n2} & 1 - a_{nn} \\ \\ \end{bmatrix} = (V_1, V_2 \; . \; . \; . \; V_n) \qquad (38\text{-}2)$$

If ($0 \leq x$ and 0 ( ) then the economic system is productive

If ($0 < P$ and 0 (V) the economic system is profitable.

Solving equation (8) given the final demand beam we find the quantity of the production beam × i.e. the determination of levels

Production XJ to meet the final demand of YI .

And solving equation (9) given 7 rays the added value of one unit for each sector we find the value of a ray

Prices P that corresponds to the V ray.

INPUT AND OUTPUT ANALYSIS (I-O-A) IS A METHODOLOGY THAT ILLUSTRATES THE INTERDEPENDENCE BETWEEN DIFFERENT SECTORS OF THE ECONOMY WITHIN A SYSTEM OF LINEAR EQUATIONS.

And outputs in general as a mathematical equation:

$Ax + y = x$ (39-2)

where A is a matrix (nn) for technical coefficients with arij elements indicating the requirements (inputs) for sector g of sector that has been normalized with respect to the overall input requirements for sector g, x is the total output beam, nor is it the final demand beam. It should be noted that Ax In equation (1) represents intermediate demand, which is used for further processing, while the final demand beam responds directly to the requirements of end-users by considering the interrelationships that exist within sectors, so that in an economy a sector may need inputs from itself and other sectors to produce its outputs, the sector's impact may not only be due to the size of the sector but also because of its dependence on the outputs of other sectors for its output as the inputs it needs.

2 - Matrix of immediate needs and mathematical formula

The beginning of the transformation of the exchange matrix of goods and service flows into technical transactions, so that any change at any level of production can be traced within the economy. If the exchanger matrix reflects all sales

Any change in one sector will affect the output of other sectors.

The good question: How can such effects be traced? Deriving a Direct Technical Coefficient Matrix from a Table

Exchangers by dividing each digit in the column) by the sum of the accompanying column Result Technical Parameters Matrix

Directly shows the decimal value of parts of one monetary unit of inputs purchased by one sector of

All sectors of the economy, to produce one monetary unit of its mathematical output if the matrix of exchanges is denoted by

Zi and dividing the value of each digit in it by the sum of its column, will produce a matrix of direct technical coefficients usually denoted by the symbol (A) and its elements (as), where the mathematical formula for it is as follows:

.(i = j = 1،2 ،.. .. n) من أجل $a_{ij} = Z_{ij} / x_j$

where: AIJ direct technical coefficient expresses the percentage of total purchases of Sector G provided by Sector A.

Reading the column as a single unit for technical transactions shows how the economic sector purchases its supplies by other economic sectors, including the sector itself. If the sector imports part of its inputs from the outside world, then the sum of direct transactions is less than one, i.e. the difference between the sum of the direct technical transactions of the sector (One (1) is the proportion of goods and services in the sector purchased from the outside world. This section is always a square matrix. While describing the accounting framework for inputs and outputs and tables of inputs and outputs Physical or cash flows between different sectors within the national economy in absolute measures, input and output coefficient matrices are used for mathematical analysis of inputs and outputs (IDA). The direct requirements matrix consists of technical parameters for each industry or product. Technical coefficients are calculated from the input and output table. The basic assumption in the analysis of inputs and outputs is based on the observation that each sector has a distinct combination of inputs for each unit of outputs Main equation No. (151) of the first chapter above in the analysis of inputs and outputs:

x = Ax + y

means that demand equals supply (input) - output). The components of the first equation are:

AX = intermediate deal between economic sectors, where A is the matrix of input coefficients in economic sectors (called the direct requirements matrix and X is the vector (ray) of the product of economic sectors. Besides the requirements from the current locally produced intermediate demand, the direct requirements matrix A can include Requirements from the medium demand for locally produced capital and the current imported intermediate demand and imported capital.

= Y final demand to economic sectors

The first equation can be rewritten, so that the total width is a function of the final request, mentioned above from the chapter

First No. (18-1)

X = (I-A)-y

where the term (I-A) is called the Leontief inverse and represents the direct and indirect cumulative use of intermediate products per unit of production that can be expanded into an infinite series of intersectoral transactions:

$$X = (I + A + A^2 + A^3 \ldots + A^{n\to\infty})Y$$

This equation is useful when calculating cumulative (retracement) effects layer by layer, i.e. initial rebound, initial bounce, second and third bounce until infinite bounce. The cumulative effects of each level are illustrated as in throwing a stone into a pool of water, and the rebound waves are formed, starting with the first major wave, then the waves continue and get smaller

in force until it fades after a period of time.

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